# MTH5102 : CALCULUS III, 2009-10
# Course Information Sheet

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## Contents of the course

This course teaches some tools that are essential for parts of the maths that you might study in the future, though not everybody will take courses that use every part of this one.

We start with some revision of key material from Calculus I and Calculus II, including the basic functions sin,cos,ln,exp, then surface integrals and volume integrals, and the basic definitions of vectors with scalar and vector products.

We continue with representing general curves in 2 dimensions via parametrisation, and calculating lengths of these curves. We also study curves represented in polar coordinates, and areas and lengths of these.

We next move on to vector calculus. This is a must for anyone wanting to do further applied maths courses, and ought to be a part of any mathematician's education. It is relevant to many situations where functions of more than one variable arise. It will in particular tell you the meaning of hieroglyphs such as $\nabla f$, div $\mathbf{f}$, curl $\mathbf{u}$ and $\nabla \times \mathbf{v}$ that you might come across in many contexts.

We continue with the extension of vector calculus into non-Cartesian coordinate systems, such as cylindrical polars and spherical polars (in general, called curvilinear coordinates).

We then study series solutions of ordinary differential equations (ODEs), with particular application to the Legendre equation and its solution in Legendre polynomials, which occurs in many important physical applications.

Next we study the method and application of Fourier series, which shows us how to represent an arbitrary function as a sum of sin and cosine functions of different periods. This approach is now of interest in many fields where data vary in space or time or both, not only in the physical applications Fourier originally had in mind.

Finally, we shall meet Laplace's equation, an important first example of a partial differential equation, and learn how to apply some basic methods of solution to some of its simpler cases. This brings together ideas from the vector calculus, differential equations and Fourier series sections.

## Teaching

The teaching for the course takes place by means of lectures, supplemented with exercise sheets and supporting examples classes. You are expected to attend lectures and examples classes, and to hand in at least six coursework questions.

### Lectures

Lectures commence on Monday 28 September , and will be at the following times and places:
Monday 9:00 – 10:00, Geography 126
Thursday 16:00 – 17:00, Physics Lecture Theatre
Friday 13:00 – 14:00, FB 240.

(Please check at `http://www.maths.qmul.ac.uk/undergraduate/current/timetable/` for the most up-to-date information).

In 2008, two-thirds of the people who failed the course had not been attending lectures, just relying on reading the notes. This works for some people but apparently not for most. So I advise you to come to lectures.

### Examples classes

Examples classes are on Wednesdays: 10AM in FB 115, 10AM in Geography 226, and 11AM in FB 102.6. Students whose surnames start with A-I should go to FB 115, and surnames J-Q to Geography 226, if possible, and surnames starting with R-Z to FB 102.6. Marked coursework from the previous Thursday handin, with its feedback, will be available at the classes: any not collected there will be available from the Maths office.

Note that the classes start in Week 2.

## Assessment

Your final mark for the course will be derived from the marks obtained in (i) coursework, (ii) the week 7 test and (iii) the end-of-year exam.

## Coursework (10%)

The exercise sheets are intended to help you to understand and learn the lecture material. As well as the marks gained, by making an effort to do the problems you will give yourself the best chance of really understanding the mathematics, and getting a good mark on the test and on the final exam.

Hand in your written solutions to the **green** box on the **basement** floor of the Mathematics Building before 4 p.m. on the deadline date indicated on the sheet (since practical jokers sometimes swap labels on the boxes around, do not believe any change to this unless it comes directly from me). Late coursework cannot be accepted, as answers will be posted on the course webpage soon after handin. Your work will be marked and returned to you at a subsequent class (see above).

The material you need for a given coursework should all have been covered before the exercise class preceding the deadline, so that you should have a chance to discuss problems in the exercise class.

The School is short of markers and to make best use of them, we are asking them to mark fewer questions but give more detailed feedback on those which are marked: so this year, **one question on each coursework sheet will be marked with a star, and this is the one you should hand in for marking** . ( All questions will be covered in the examples classes, and model solutions will be posted on the website after the hand-in deadline. )

There will be **nine courseworks** in total, with hand-in deadlines in weeks 2,3,4,5, 8,9,10,11,12; your **best six** will count towards your final mark.

### Week 7 test (10%)

There will be a written test of 40 minutes duration under exam conditions in week 7. The exact time will be announced in due course. If timetabling allows, I will hold a revision lecture in week 7 before the test. Also, there will be no coursework hand-in in Week 6, so the Week 6 class will be revision e.g. of previous year's tests.

If you are absent from the test, you will get a mark of zero unless you provide a medical certificate or other documentation which satisfactorily explains your absence. The test will be of a multiple choice format similar to those used in the last three years: some past tests and answers are posted on the web page. The test will cover material up to week 6 and be at the level of section A of old exams.

### End-of-year examination (80%)

The end-of-year written exam (held in the usual college examination period) will test material covered at any time during the course. The date of the exam will not be known until the college examination timetable is published.

## Syllabus

The approved syllabus is:

1. Arc-length of plane curves: length of a parametric curve, length of a curve $y = f(x)$. Length of the circumference of a circle, ellipse. Area and length in polar coordinates.
2. Vector fields, line, surface and volume integrals.
3. Grad, div and curl operators in Cartesian coordinates. Grad, div, and curl of products etc. Vector and scalar forms of divergence and Stokes's theorems. Conservative fields: equivalence to curl-free and existence of scalar potential. Green's theorem in the plane.
4. Orthogonal curvilinear coordinates; length of line element; grad, div and curl in curvilinear coordinates; spherical and cylindrical polar coordinates as examples.
5. A first look at Legendre polynomials.
6. Fourier series: full, half and arbitrary range series. Parseval's Theorem.
7. Laplace's equation. Uniqueness under suitable boundary conditions. Separation of variables. Two-dimensional solutions in Cartesian and polar coordinates. Axisymmetric spherical harmonic solutions.

## Lecture Notes

We neither sell nor give out printed copies of prepared notes for this course. However, I shall post notes on the Web pages for the course. You are welcome to download/print/photocopy these, in whole or in part, to assist you with studying this course – indeed, we encourage you to do so.

## Textbooks

The online notes should mean that you do not find it necessary to buy a textbook. However, you may need to consult one.

The textbook for Calculus I and II is “Thomas’ Calculus, 11th edition”, originally by G.B. Thomas, revised by M.D. Weir, J. Hass, and F.R. Giordano (Addison-Wesley, 2005): ISBN 0321243358, College library QA303 WEI (in Short Loan Collection). Chapters 15 and 16 cover the first part of this course reasonably well: Chapter 15 will be revision as this was covered in Calculus II.

Other books covering that material are as follows. The book used in Geometry I for vectors is “Vectors in 2 or 3 dimensions” by A.E. Hirst (Elsevier 1995). The advice from those who gave the course before me is that the best book for the vector calculus part of the course is: “Vector Calculus” by P.C. Matthews (Springer, 1998): ISBN 3540761802, College Library QA433 MAT. They recommend it highly, and say it is available in paperback. You many also find M.R. Spiegel’s “Vector Analysis” (Schaum Outlines, McGraw-Hill) and S. Simons “Vector Analysis for mathematicians, Scientists and Engineers” (Pergamon) helpful. Further appropriate texts can be found in the library with ‘calculus’, especially ‘vector calculus’, or ‘vector analysis’ in the title.

There are many books in the library sections on differential equations (QA 372), partial differential equations (QA 374), Fourier series (QA 404) and Laplace’s equation (QA 404.7, under the title “potential theory”) which cover parts of the rest of the course. Most however go well beyond what we will study, so the lecture notes are probably the best starting point. But as backup reading, consider the following:

For the series methods, the text recommended for the Differential Equations course: J. Polling, A. Boggess and D. Arnold, “Differential Equations”, Pearson 2006.

For this, Fourier series and the Laplace equation results we shall study, try P.V. O’Neill “Advanced Engineering Mathematics” (the Laplace material is under the Boundary Value headings in chapter 13) or Kreysig’s similarly titled book, both in TA 330. [The advantage of using methods books for engineers or scientists for this course is that they are more oriented towards solving problems than proving theorems.] The library has multiple copies of these books, and the ones by Farlow, Stroud, and Wylie and Barrett mentioned below.

Others in the differential equations section which may be useful are by Coddington and Levinson, e.g. page 132, Braun and Ince; in the PDE section, by Berg and McGregor, DuChateau and Zachmann, and by Spiegel; in the Fourier section, by Bolton; and in the methods books section by Farlow, Jeffrey, Riley, Stroud, and Wylie and Barrett.

## Prerequisites

The formal pre-requisites are Calculus II (which itself requires Calculus I) and Geometry I. In practice you need to know the following (let me know about anything listed here which was not covered, as it should have been, in the relevant course). Those items asterisked will be briefly reviewed in the lectures: references to Thomas’ Calculus are given where relevant.

*From Calculus I*

Basic differentiation and integration (Thomas, chapters 3 and 5)

Trigonometric functions (mainly $\cos x$ and $\sin x$). $*$. (Thomas 1.6, 3.4, 8.4 and 8.5)

Log, exp and hyperbolic functions (cosh and sinh etc). $*$. (Thomas 7.2, 7.3 and 7.8)

Integration by parts. (Thomas, section 8.2)

*From Calculus II*

Infinite sequences and series, especially power series. (Thomas 11.1, 11.2, 11.7)

Basic idea of limits and continuity in the *xy*-plane. (Thomas chapter 2)

Partial derivatives. The Chain Rule. (Thomas 14.3, 14.4)

Directional derivatives and gradient vectors. Tangent planes and differentials. $*$. (Thomas 14.5, 14.6)

Double integrals. Triple integrals. Substitutions in multiple integrals. $*$. (Thomas 15.1, 15.2 (Areas only), 15.3, 15.4, 15.7)

*From Geometry I*

Vectors in 2-space and 3-space, expressed as $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ or as row or column vectors. Addition of vectors. Length of vectors. $*$.

Vector and cartesian equations of a straight line in $\mathbb{R}^2$ and $\mathbb{R}^3$ $*$.

Scalar multiple and scalar product of vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$. Cartesian equation of a plane in $\mathbb{R}^3$ $*$.

Vector products in $\mathbb{R}^3$ $*$.

Cartesian equations of ellipse, parabola, hyperbola. $*$.

# Problems

Please feel free to see me to discuss any problems that you are having with the course. Your problem may well be shared by others in the class, in which case we can spend more time in lectures dealing with the problem area. Ask during the lectures, or come and see me in my "office hours": my room number is on the front of this handout and my office door bears my timetable (including any special changes to office hours). My timetable is also on my Web page at `http://www.maths.qmul.ac.uk/~wjs` . Normally office hours will be at the same time each week, and I will be in my office then to answer any questions that you might have. Please check outside my room or on my Web page for details of office hours.

If you are absent for any significant length of time during the course (due to illness etc.), please inform me as soon as possible and fill out an extenuating circumstances form and get the Senior Tutor, Thomas Prellberg, to sign it. It may be possible to take these things into account when calculating the final mark, but we need to know.

# KEY OBJECTIVES of the course

The student should

1. Be able to do simple line and surface integrals. (E.g. Evaluate $\int \mathbf{F}\cdot d\mathbf{r}$ for a given vector field, with the path given in either parametric or non-parametric form.)

2. Understand three-dimensional cartesian, cylindrical, and spherical polar coordinates geometrically, and be able to express lines, surfaces, and volumes in coordinate or vector notation as appropriate.

3. Be able to do simple manipulations involving gradient, divergence, and curl, and understand their geometrical/physical meaning.
4. Understand Stokes' theorem and the divergence theorem and be able to do simple problems applying these.
5. Have a basic understanding of the Legendre equation and its solution in Legendre polynomials.
6. Know the important properties of Fourier series and be able to compute coefficients.
7. Understand the variable-separation technique for PDEs and be able to solve simple problems with Laplace's equation in (at least) 2D Cartesian coordinates.

## Examination

It is likely that the examination will have similar format as that for 2008-09, i.e. nine or ten questions, and all should be attempted. Further details of the examination will be provided in due course on the College website.