

# A three-species model explaining cyclic dominance of pacific salmon

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## Abstract

The four-year oscillations of the number of spawning sockeye salmon (*Oncorhynchus nerka*) that return to their native stream within the Fraser River basin in Canada are a striking example of population oscillations. The period of the oscillation corresponds to the dominant generation time of these fish. Various - not fully convincing - explanations for these oscillations have been proposed, including stochastic influences, compensatory fishing, or genetic effects. Here, we show that the oscillations can be explained as an attractor of the population dynamics, resulting from a strong resonance near a Neimark Sacker bifurcation. This explains not only the long-term persistence of these oscillations, but also reproduces correctly the empirical sequence of salmon abundance within one period of the oscillations. Furthermore, it explains the observation that these oscillations occur only in sockeye stocks originating from large oligotrophic lakes, and that they are usually not observed in salmon species that have a longer generation time.

*Keywords:* cyclic dominance, population oscillations, single-generation cycles, Neimark Sacker bifurcation, strong resonance

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## 1. Introduction

In many ecological systems distinct population oscillations are known, with their specific dynamical characteristics often captured by simple generic models. Among these are the spatial synchronisation of the lynx-hare oscillations in Canada [1, 2, 3], the chaotic oscillations of boreal rodents in Fennoscandia [4], or the cyclic outbreak dynamics of the spruce budworm [5, 6]. The four-year oscillations of sockeye salmon (*Oncorhynchus nerka*) in the Fraser River basin in Canada are another well-documented example of such large-scale population oscillations [7, 8, 9]. They are a typical example of single-generation cycles [10, 11]. Every fourth year (which is the dominant generation time) the abundance of these fish is at very high levels, reaching several million fish in some spawning populations, but drops to numbers between several hundred and a few ten thousand individuals in the following years (hence the term *cyclic dominance*). Different stocks can have their population maximum in different years (Fig. 1 and Appendix). The oscillations were reported as early as the 19th century and are evident for instance in the extremely high catches by fisheries every fourth year [12]. This both economically and conservationally important phenomenon has been ascribed either to transient effects or to stochastic influences [13], to depensatory predation [14], to fishing [15], or to genetic effects [16, 17], but all of these explanations are still not fully convincing [16, 9].

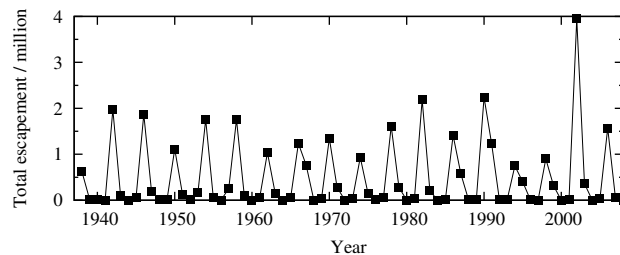


Figure 1: **Empirical time series of sockeye salmon abundance.** Total escapement (number of male and female adults that escape the fisheries) of the late Shuswap Lake run. Shuswap lake is one of the most important rearing lakes of juvenile sockeye salmon. Data for 5 other lakes are provided in the Appendix.

The sockeye salmon return to spawn in their native stream or lake in late summer and then die, which means that the salmon generations do not overlap. The hatched fry migrate downstream in the following spring and feed for one season in large freshwater lakes, before they migrate to the ocean, where they spend the next two and a half years. The carcasses of the adult salmon are decomposed and provide an important phosphorus input into the rearing lakes of the fry. Since the oscillations of sockeye salmon originating from different lakes are not in phase, we have clear evidence that the relevant processes causing the phenomenon of cyclic dominance take place in the rearing lakes rather than in the ocean.

## 2. Model and results of computer simulations

It is our aim to develop a generic model capturing only the essential mechanisms required for the occurrence of cyclic dominance. As such, the model is kept very simple and is not designed to quantitatively predict the population dynamics of all species in the corresponding lake ecosystems.

The model uses standard continuous population dynamics equations for sockeye fry,  $s_n(t)$ , their predator (e.g. rainbow trout),  $p_n(t)$ , and their zooplankton food (mainly daphnia),  $z_n(t)$ , during the growth season from spring ( $t = 0$ ) to fall ( $t = T$ ) in year  $n$ , combined with a rule for calculating the three population sizes at the beginning of the next season as a function of the population sizes at the end of the previous season(s). The sockeye fry population at the end of a season gives rise to the number of spawning adults 3 years later, which in turn determines the number of sockeye fry in the following spring, and represents a nutrient input that affects the carrying capacity of the zooplankton. A small fraction of the salmon stays in the ocean for one more year and mature at age 5, thus causing a mixing between the four brood lines of a spawning population. An even smaller fraction matures at age 3, but since these fish are predominantly small-sized males [9] that do not influence the number of fertilised eggs, they are neglected in this study.

The general structure of the model, which combines continuous population dynamics during the growth season with a discrete update scheme capturing the seasonally driven migration and reproduction of the species of interest, has also been applied in a modelling study of the dynamics of fish preying on zooplankton [18]. Here, however, we have chosen a much simpler form of the continuous dynamics which is sufficient to generate cyclic dominance. It is given by the following equations of motion:

$$\begin{aligned}
\frac{d}{dt} s_n(t) &= \lambda \cdot a_{sz} \frac{z_n(t) \cdot s_n(t)}{1 + c_s \cdot s_n(t) + z_n(t)} \\
&\quad - a_{ps} \frac{s_n(t) \cdot p_n(t)}{1 + c_p \cdot p_n(t) + s_n(t)} - d_s \cdot s_n(t) \\
\frac{d}{dt} z_n(t) &= z_n(t) \cdot \left( 1 - \frac{z_n(t)}{K_n} \right) \\
&\quad - a_{sz} \frac{z_n(t) \cdot s_n(t)}{1 + c_s \cdot s_n(t) + z_n(t)} \\
\frac{d}{dt} p_n(t) &= \lambda \cdot a_{ps} \frac{s_n(t) \cdot p_n(t)}{1 + c_p \cdot p_n(t) + s_n(t)} - d_p \cdot p_n(t)
\end{aligned} \tag{1}$$

The variables  $s_n$ ,  $z_n$ , and  $p_n$  are biomass densities that are made dimensionless. The parameters in these equations and their numerical values used in the computer simulations are as follows.  $\lambda = 0.85$  denotes the assimilation efficiency of ingested prey biomass of carnivores [19]. The maximal per unit mass ingestion rates of salmon,  $a_{sz} = 10$ , and predators,  $a_{ps} = 1.6$ , scale allometrically with bodymass according to a power law with exponent  $-1/4$  [20]. We assume the predators to be approximately 1500 times heavier than the sockeye fry, which fixes the ratio  $a_{sz}/a_{ps}$ . The parameters  $d_s = 1$  and  $d_p = 0.16$  represent biomass loss due to respiration and mortality; they also scale allometrically with body mass. The feeding terms include saturation at high prey densities, and a predator interference term in the denominator with interference strengths set to  $c_s = 1$  and  $c_p = 0.2$ . This Beddington functional response [21] has been chosen since it describes consumer-resource interactions more accurately than the more frequently used Holling type II form [22]. Similar predator interference terms are also used by other modellers [23].

The matching conditions used to determine the biomasses of the species at the beginning of the next

58 season from their values at the end of the previous season(s) are given by

$$\begin{aligned}
 s_{n+1}(0) &= \gamma((1 - \epsilon)s_{n-3}(T) + \epsilon s_{n-4}(T)) \\
 z_{n+1}(0) &= K_{n+1} \\
 p_{n+1}(0) &= p_n(T),
 \end{aligned}
 \tag{2}$$

with  $\epsilon$  the proportion of surviving sockeye that return to their native lakes at the age of 5 to spawn  
 60 and die. The parameter  $\gamma$  summarises survival from smolt to adult fish (including ocean survivability  
 and loss to fisheries), spawning success, and egg to fry survival. The various factors are estimated  
 62 from empirical data [15, 24, 25] and yield  $\gamma = 0.4$ .

According to empirical observations [16], the zooplankton level at the end of one year has no effect  
 64 on the following year. However, its carrying capacity  $K_{n+1}$  is a function of the nutrients provided by  
 the adult salmon and thus a function of the initial fry biomass of that year [26]:

$$K_{n+1} = K_0 + \left( \kappa \frac{s_{n+1}(0)}{\kappa_0 + s_{n+1}(0)} \right). \tag{3}$$

66 (For a review on the importance of salmon-derived nutrients for freshwater ecosystems, see [27] and  
 citations therein.) Consumer dependent resource productivity is certainly a rather uncommon phe-  
 68 nomenon. In the lake ecosystems under consideration, it provides an important positive feedback  
 to the system, but it does not seem to be crucial for the mathematical mechanism underlying the  
 70 oscillations [28]. The three parameters determining the carrying capacity  $K_n$  of the zooplankton in  
 year  $n$  are set to  $K_0 = 15$ ,  $\kappa = 10$ , and  $\kappa_0 = 2$ . The numerical values of these parameters are difficult  
 72 to determine from empirical data. However, since the nutrient concentration in the brood lakes nearly  
 doubles in years following dominant sockeye returns compared to years following non-dominant sock-  
 74 eye returns [26], it seems reasonable that the constant fraction of the carrying capacity,  $K_0$ , and the  
 maximum of the variable fraction,  $\kappa$ , are of the same order of magnitude.

76 Fig. 2a shows a time series of the biomass of the sockeye fry at the end of the growth season. The first  
 300 years are cut off to show only the stable periodic oscillation with one strong year followed by one  
 78 intermediate year and two weak years, just as in the empirical data of most sockeye stocks showing  
 cyclic dominance (Fig. 1 and Appendix). When the simulation parameters are chosen differently (e.g.,  
 80  $c_s = 0.2$ ), the difference between the strong and the intermediate year may become less pronounced.  
 By additionally increasing  $K_0$  and  $c_p$ , their order may even become reversed. This is also observed in  
 82 the empirical data of some stocks.

When a parameter is varied, for instance the constant fraction of the zooplankton carrying capacity,  
 84  $K_0$ , the dynamical pattern may change. Fig. 3 shows the biomass  $s_n(T)$  of the sockeye fry at the  
 end of the season for different values of  $K_0$ , from year  $t = 5000$  to year 5200. For small  $K_0$ , all  
 86 sockeye lines are equally strong (only one point is visible for each value of  $K_0$ ). This means that  
 the dynamics reaches a fixed point and that there is no cyclic dominance. With increasing  $K_0$ , the  
 88 fixed point eventually becomes unstable. First a bifurcation to quasiperiodic behaviour occurs (the  
 attractor contains infinitely many points for each value of  $K_0$ ), and then the frequency of the oscillation  
 90 becomes locked at 4. Now only four points are visible for each value of the bifurcation parameter,  
 indicating periodic oscillations.

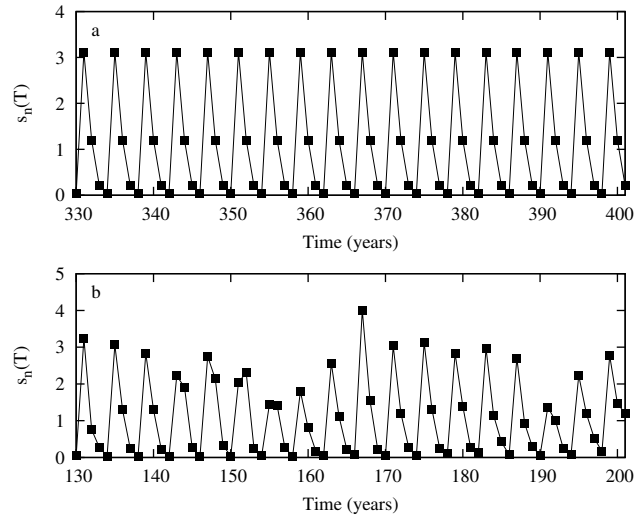


Figure 2: **Simulated time series.** Attractor of sockeye salmon dynamics generated by computation of the time-continuous model, showing oscillations with period 4 and the same pattern in the sequence of salmon abundance as in the empirical data. Only the biomasses at the end of each growth season are shown. a: deterministic model, b: dynamics with up to 50 percent fluctuations in the survivability of the sockeye salmon in the ocean (proportional to  $\gamma$ ), see *Discussion*. Parameter values for these simulations are as explained in section 2.

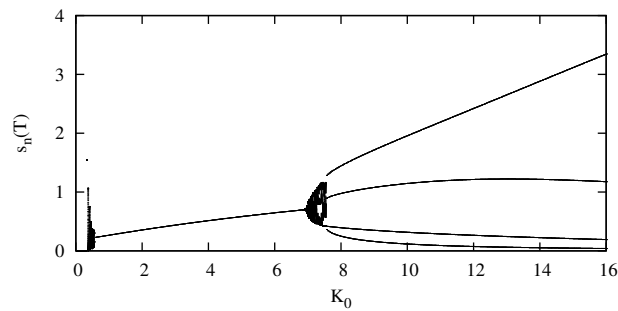


Figure 3: **Bifurcation diagram of the biomass of the sockeye fry at the end of the season.** The bifurcation parameter is the constant fraction of the zooplankton carrying capacity  $K_0$ . For each value of  $K_0$ , 200 consecutive data points are plotted.

92 **3. Bifurcation analysis**

In order to understand and interpret these observations, we first note that the continuous population  
 94 dynamics during the season, together with the matching conditions applied between two seasons, can  
 be viewed as a discrete map (a so-called Poincaré map), giving the sockeye and predator biomasses  
 96 at the end of one year as function of the biomasses at the end of the previous years.

To obtain this map, one first has to integrate the dynamical equations over one season, giving  $s_n(T)$   
 98 and  $p_n(T)$  as a function of  $s_n(0)$  and  $p_n(0)$ . The zooplankton can be eliminated since the initial  
 condition of the zooplankton depends on  $s_n(0)$  only. Next, one expresses  $s_{n+1}(0)$  and  $p_{n+1}(0)$  as a  
 100 function of  $s_{n-3}(T)$ ,  $s_{n-4}(T)$  and  $p_n(T)$ , using the matching conditions.

The mechanism which generates the population oscillation is based on the nature of the instability of  
 102 the stationary state  $s_n(T) = s^*(T)$ ,  $p_n(T) = p^*(T)$  of our system. The corresponding bifurcation can  
 be investigated in terms of a linear stability analysis. Close to the fixed point, the dynamics can be  
 104 approximated by linear terms. Denoting the distance of the biomasses from their fixed point value by  
 $\delta s_n = s_n(T) - s^*(T)$  etc, the linear approximation of this map has the form

$$\begin{pmatrix} \delta s_{n+1} \\ \delta s_n \\ \delta s_{n-1} \\ \delta s_{n-2} \\ \delta s_{n-3} \\ \delta p_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & m_s & \epsilon_1 & -a \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & b & c & m_p \end{pmatrix} \begin{pmatrix} \delta s_n \\ \delta s_{n-1} \\ \delta s_{n-2} \\ \delta s_{n-3} \\ \delta s_{n-4} \\ \delta p_n \end{pmatrix} \quad (4)$$

106 with positive parameters  $m_s, m_p, a, b, c, \epsilon_1$ . The general structure of the matrix in Eq. (4) is determined  
 only by the matching conditions and is valid for any time-continuous model applied during the seasons.  
 108 The equations of motion of the latter only determine the numerical values of the parameters in Eq.  
 (4).

110 The first line of the matrix describes the influence on the sockeye fry of year  $n + 1$  of the sockeye fry  
 of year  $(n - 3)$  and  $(n - 4)$  (which are the parents of the fry in year  $(n + 1)$ ), and of the predator  
 112 population.  $m_s$  and  $\epsilon_1$  are positive, since more parents imply more offspring.  $-a$  is negative, since  
 more predators imply less fry. The other nontrivial line of this matrix, the last line, describes the  
 114 influence on the predators of year  $(n + 1)$  of the sockeye fry of year  $(n - 3)$  and  $(n - 4)$  (which are  
 the parents of the predator's food), and of the predator population in the previous year. All three  
 116 parameters are positive, since more food implies more predator growth and since more predators in  
 one year give rise to more predators in the next year.

118 The eigenvalues of this matrix determine the nature of the dynamics near the bifurcation. When  
 all eigenvalues have an absolute value smaller than 1, the fixed point is stable, and the dynamics  
 120 converges to this fixed point. When the absolute value of one or more eigenvalues is larger than 1,  
 the fixed point is unstable, and the dynamics approaches a different attractor.

122 In order to understand the dynamics resulting from this matrix, we first consider the case that the  
 matrix elements  $\epsilon_1, b$ , and  $c$  vanish. This means that all salmon return at age 4, and that the trout  
 124 have a good choice of other food so that their dynamics is independent of that of the salmon fry. In

this case the eigenvalues of the matrix are  $m_p$ , 0, and the four fourth roots of  $m_s$ . Since the four
   
 126 salmon lines are independent from each other in this case, the sequence  $\delta s_n$  has trivially the period
   
 4 and simply iterates the initial four values, with an amplitude decreasing for  $m_s < 1$  and increasing
   
 128 otherwise (and with the trout being independent of the salmon). When  $m_s$  is increased from a value
   
 smaller than 1 to a value larger than 1, all four eigenvalues  $m_s^{1/4}$  cross the unit circle simultaneously,
   
 130 and the fixed point becomes unstable. This degeneracy is lifted when the parameters  $\epsilon_1$ ,  $b$ , and  $c$  are
   
 made nonzero. As long as these parameters are not large, one can expect the four main eigenvalues
   
 132 to remain close to the real and imaginary axis, respectively, implying a (possibly damped) oscillation
   
 with a period close to 4.
   
 134 The type of bifurcation that occurs when the fixed point becomes unstable depends on which eigenvalue
   
 crosses first the unit circle as a control parameter is increased. In order to determine the type of the
   
 bifurcation, we evaluated the parameters of the matrix numerically from our computer simulations
   
 136 of the time continuous model when going through the bifurcation (i. e., when  $K_0$  increases from
   
 6.4 to 7.5), and we calculated the eigenvalues  $\lambda$  of the matrix. Figure 4 shows that the bifurcation is
   
 138 caused by a pair of complex conjugate eigenvalues crossing the unit circle, indicating a Neimark Sacker
   
 bifurcation (the discrete version of the Hopf bifurcation). Since the dominant eigenvalues (depicted
   
 140 with the symbol +) are close to  $\pm i$ , the period of the resulting oscillation is close to 4. In the linear,
   
 time discrete model (Eq. (4)), the bifurcation is mainly driven by the parameters  $m_s$  and, to a minor
   
 142 extent,  $a$ , which correspond to reproduction of the four-year old sockeyes and predation by the rainbow
   
 144 trouts, respectively. The fixed point becomes unstable when either of these parameters increases.

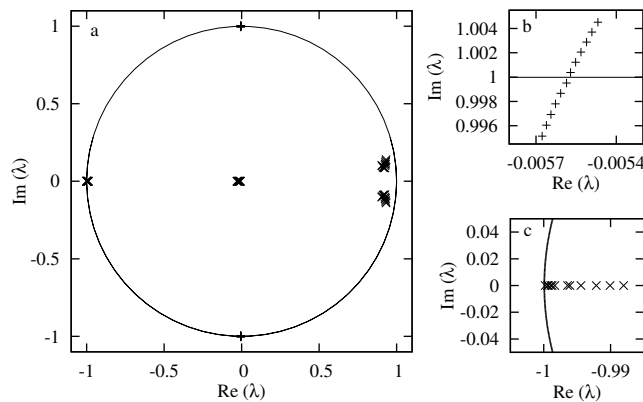


Figure 4: **Eigenvalues of the matrix from Eq. (4), as obtained from a linear stability analysis of the time-continuous model.**  $K_0$  increases from 6.4 to 7.5 as the eigenvalues move outwards. The zoom shows that the eigenvalues close to the imaginary axis (+) are the first ones to cross the unit cycle.

Now, it is known from the theory of bifurcations that if the period at such a bifurcation is close to
   
 146 4, there occurs a strong resonance, which means that the period becomes locked exactly at the value
   
 4 not far beyond the bifurcation point. In contrast to conventional mode locking that gives rise to
   
 148 Arnold tongues, a strong resonance is due to nonlinearities that are of the same order as the leading
   
 nonlinearity (which ensures the stability of the quasiperiodic orbit in the first place), and frequency
   
 150 locking therefore occurs over a much wider range of parameters [29].

The two complex conjugate eigenvalues are the first ones to cross the unit circle when  $a$  and  $\epsilon_1$  are

152 small non-negative numbers and when  $m_p$  is well below 1. Since  $\epsilon_1$ ,  $b$ , and  $c$  are small compared to  
153  $m_s$ , the period of the resulting oscillation remains close to 4, and the locking at period 4 therefore  
154 occurs not far beyond the bifurcation. When these conditions are not met, the unit circle is typically  
first crossed at -1, and a period-doubling (or *flip*) bifurcation occurs.

#### 156 4. Discussion

The four-year oscillations of sockeye salmon in the Fraser River are a typical example of single-  
157 generation cycles (as opposed to predator-prey cycles [10]), a phenomenon that has also been exten-  
158 sively studied in the context of, e.g., host-parasitoid [30] or daphnia-zooplankton [31] systems. In the  
159 latter case, individuals of different age or size of a consumer species directly compete with each other  
160 for a resource, while the biological situation in the present study is somewhat different: The salmon  
161 fry of different brood lines do not compete with one another directly, since they populate the rearing  
162 lakes at distinct, non-overlapping times. The interaction between the brood lines is mediated by the  
163 long-living predators, which have a net effect on the salmon that is similar to competition (apparent  
164 competition). In contrast to other models that produce single-generation cycles, the cycle period 4  
165 observed in our system is not identical to the mean generation time, which is larger than 4 years, and  
166 not all salmon mature and reproduce at the same age. The locking of the period at 4 is due to a  
167 strong resonance, a phenomenon which plays no role in the cited models.

The three-species model analysed in this study is in principle also capable of showing conventional  
169 predator-prey cycles. If the parameters were chosen such that the complex conjugate pair of eigen-  
170 values of the matrix in Eq. (4) that is close to the real axis at +1 is the first to cross the unit circle,  
171 oscillations with a period much larger than four years occur and the year-to-year variation between  
172 brood lines is rather small (results not shown). However, this type of oscillations is not seen in the  
173 sockeye populations rearing in the Fraser River basin.

The considerations in the previous section lead to two basic conditions under which strong resonance  
174 and thus period-four generation cycles can be observed: First, an increase in the number of spawning  
175 salmon must lead to a sufficiently strong increase in this number four years later (i.e.  $m_s$  must be  
176 large enough). If the number of fry migrating to the ocean is dominated by other factors, such as a  
177 strongly limited carrying capacity for the fry, the population will be at a fixed point rather than on  
178 the oscillating side of the bifurcation. Second, the four salmon lines must be coupled in order for the  
179 Neimark Sacker bifurcation to occur, rather than a period doubling bifurcation. In our model, this  
180 coupling is due to a fraction of sockeye returning at age 5 instead of age 4, and, more importantly,  
181 due to the predator being sufficiently strongly coupled to sockeye dynamics.

182 Previous studies of salmon dynamics, used in salmon management, are based on the Ricker model  
183 [7, 24] or the Larkin model [14, 32, 33]. Because both models have a strongly limited carrying capacity  
184 for the fry, and because they do not include explicitly the predator dynamics, the only bifurcation  
185 occurring in those models is the flip bifurcation. Building on both modelling approaches, Myers et al.  
186 [13] have demonstrated that an unstable (decaying) period-4 oscillation can be excited by stochastic  
187 driving. However, it remains unclear over how long the oscillations can be maintained in that study  
188 without a change in the phase. Furthermore, that stochastic model produces only episodically the  
189

typical sequence of a dominant, a subdominant and two weak years described in section 2 and seen  
192 in our simulations, Fig. 2).

In principle, the coupling between the four salmon lines can also occur through the food of the sockeye  
194 fry. However, there is no empirical evidence that there is a negative effect of a strong sockeye year  
on the daphnia population in the following year that limits the growth of the sockeye fry in that year  
196 [16].

The two conditions for a strong resonance of period 4 fit together with the empirical observation that  
198 it occurs only in large oligotrophic lakes, such as those of the Fraser River basin. The smaller and  
ultra-oligotrophic lakes in the outer coast regions of British Columbia do not show these oscillations.  
200 Their nutrient content is neither large enough to raise large fry numbers, nor to allow for salmon  
predators to become strong. On the other hand, cyclic dominance cannot be expected in nutrient-  
202 rich lakes, because the spawning adults would not be an important nutrient input, and because the  
coupling to the predator would not be strong enough in a situation where there were more predator  
204 species and more prey species for these predators.

Since there are large fluctuations in the proportion of fry that survive to return to their nursery lake,  
206 we included noise in the matching condition for the sockeyes, Eq. (2), in order to determine with  
how much noise superimposed on the deterministic dynamics the period-4 oscillation can persist. Fig.  
208 2b shows a data series generated with 50 percent noise in ocean survivability superimposed to the  
deterministic dynamics. The oscillation is still clearly visible, although the system has a phase shift  
210 every 300 years on average. With less noise, the phase shift occurs less often, and with 100 percent  
noise, the cyclic dominance vanishes in our simulations. When a large perturbation acts only for a  
212 limited time, the oscillation quickly recovers afterwards. In fact, the recovery following the blocking  
of the Fraser River migration routes early in the last century, can be seen in the non-dominant brood  
214 lines of the Shuswap stock in Fig. 1 and in the data of some of the stocks presented in the Appendix.

Unfortunately, the year 2009 has seen another large perturbation, with most Fraser sockeye expected  
216 to return that year not surviving in the ocean, so that the expected strong escapement did not occur.

The age composition of sockeye stocks was also found to strongly influence the potential of the system  
218 to show a strong resonance. The fraction of adult sockeyes that returns at the age of 5 years instead  
of 4 years,  $\epsilon$ , was set to 0.1 in the simulations and we could show that the resonance appears for  
220  $0 < \epsilon \leq 0.3$ , but for very small values of  $\epsilon$  ( $< 0.02$ ) the non-dominant lines disappear completely  
with increasing  $K_0$ . The resonance condition was best met at  $\epsilon \approx 0.2$ , where the resonance occurred  
222 nearly directly after the Neimark Sacker bifurcation. The parameter  $\epsilon$  is also measured in the real  
populations [34]. For the sockeye salmon populations of the Fraser River it is approximately 0.08,  
224 while for the less productive outer coast lakes of British Columbia (where cyclic dominance is not  
observed) it is between 0.56 and 0.76.

Some of the sockeye populations of the Bristol Bay area (Alaska), most notably the Kvichak River  
226 stocks, also exhibit strong oscillations [35, 36] despite a broader distribution of the age at spawning  
than in the Fraser River stocks [37, 38]. However, the oscillations are not as regular, with maxima of  
228 the populations occurring every fourth or fifth year. This is consistent with our model, as it indicates

230 a quasiperiodic oscillation rather than a fixed oscillation period associated with a strong resonance.  
Our results do not rule out additional mechanisms such as compensatory fishing [15] or genetic effects  
232 [17], which would reduce the population sizes of weak lines to values smaller than those resulting from  
our model. However, these additional assumptions are not needed to explain the occurrence of cyclic  
234 dominance in the first place.

### Acknowledgements

236 This work was supported by the German Research Foundation (DFG) under contract number Dr300/7  
and Br2315/9-1. Kim Hyatt, Jeremy Hume, and Carl Walters commented on an earlier version of  
238 this manuscript. Empirical data were kindly provided by Tracy Cone (escapement data) and by Kim  
Hyatt (age composition of the sockeye stocks). This collaboration started at the Complex Systems  
240 Workshop in Fairbanks, Alaska, 2007. The 2008 Cultus Lake Modelling Workshop, which had an  
important impact on this project, was hosted by Fisheries and Oceans Canada.

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326 **Appendix: Escapement data for sockeye stocks showing cyclic dominance**

Many spawning stocks of sockeye salmon in the upper Fraser River basin are not stationary but have  
328 been dramatically increasing in size over the last decades (e.g. Figure A1 a, b, and d), following a  
massive disturbance of the stocks at the beginning of the 20th century. Nevertheless, the oscillatory  
330 pattern has clearly emerged again.

In some stocks (most notably the Bowron Lake stock, Figure A1 c), cyclic dominance is observed only  
332 episodically. This dynamical pattern may appear when the system is close to the bifurcation point.

The sockeye salmon stock of Seton Lake (Figure A1 e) also displayed cyclic dominance, but for nearly  
334 two decades (five cycles), the dominant line is followed by the weakest line instead of the sub-dominant  
line. In this stock, cyclic dominance broke down in the early 1990's.

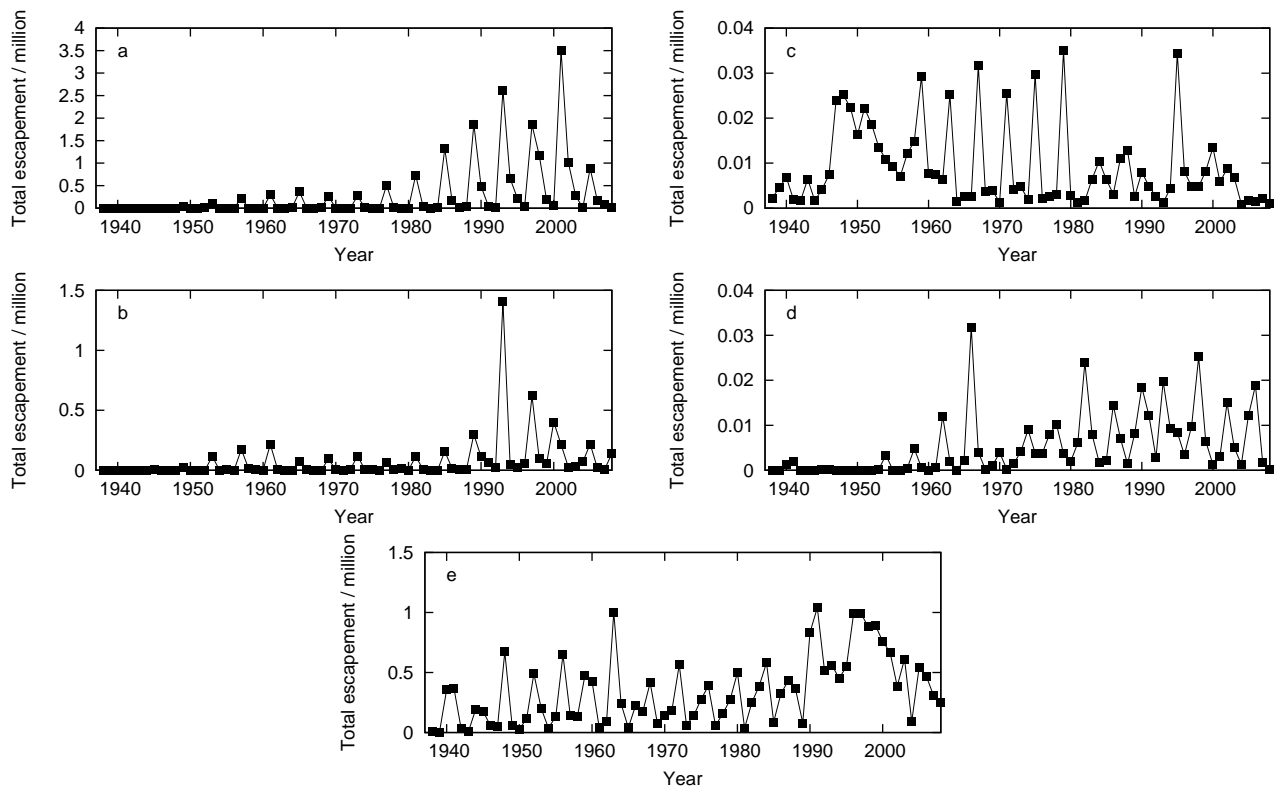


Figure A1: Escapement data of for sockeye spawning stocks in the Fraser River basin. a: Quesnel Lake, b: Stuart Lake, c: Bowron Lake, d: Seton Lake, e: Chilko Lake.