# Preface

Dynamical aspects of nonlinear systems, and in particular dynamical behaviour far from equilibrium, is one of the few areas in theoretical physics which has been very active during the last decade. Whereas the early approaches in Nonlinear Dynamics have dealt with systems of few degrees of freedom the concepts developed in this context have been, meanwhile, applied fruitfully for the investigation of collective behaviour in many particle physics. As a very prominent and important physical example we consider the glass transition and low temperature properties in general. Here those mechanisms which generate complex aperiodic or disordered structures and the interaction between static disorder and dynamical processes are at the centre of interest. In that context the concepts of Nonlinear Dynamics have proven to be of considerable relevance in condensed matter physics.

So far such aspects have not been covered in a consistent way by the existing literature, in particular as far as graduate textbooks are concerned. With the present monograph we intend to fill such a gap. Each chapter is written by an expert in his field. But in contrast to the original literature and conference proceedings the material is presented in such a way that it is accessible for graduate students. With the present monograph we intend that the state of the current research at the borderline between Nonlinear Dynamics and disordered systems proliferates among different disciplines of natural science in order to emphasise general mechanism, for the generation of complex structures.

From the general point of view the whole subject can be grouped in three parts which show, however, numerous links between the different topics. All aspects are illustrated by experimental application, although our special focus is on theoretical concepts.

I Pattern Formation and Growth Phenomena
Growth processes are typically related with pattern formation, instabilities, and complex dependence on external parameters. These phenomena are endemic for nonequilibirum pattern formation, e.g. the kinetics of ordering processes, the formation of amorphous or liquid phases, crystal growth, electrochemical deposition, erosion, or general front dynamics and surface growth processes. From the theoretical point of view reaction–diffusion systems, nonlinear transport equations, or stochastic field the-

ories are prominent models describing the afore mentioned physical phenomena. On a mesoscopic scale these models permit the identification of the elementary building blocks of complex patterns as well as the dynamical interaction of these entities. The resulting dynamical morphologies of surfaces and fronts cover ordered, fractal, and disordered structures. General order and selection principles are of particular interest, as they are condensed e.g. in group theoretical considerations and universal scaling laws.

II Dynamics of Disordered Systems
The generation of disordered structures as it can be found in physics e.g. in supercooled liquids and glasses constitutes a quite prominent example for pattern formation. The complex dynamics of these systems is governed by a whole spectrum of different time scales as well as by the occurrence of spatially confined ordered structures. Whereas the first feature may be responsible for self generated structural disorder as visible e.g. in the glass transition, the second feature can generate spherically periodic local order. From the theoretical perspective such features can be characterised by quantities borrowed from the physics of static disorder as well as by quantifiers developed in the context of Nonlinear Dynamics. From such a point of view the properties of glasses and other aperiodic systems constitute a bridge between complex dynamical behaviour and the dynamics of disordered systems.

III Complex and Chaotic Behaviour
For the investigation of complex chaotic behaviour the notions and concepts of Nonlinear Dynamics have been adapted recently to the peculiarities of systems with a large number of degrees of freedom, for instance in terms of Lyapunov and dimension spectra. Such approaches can be applied for the analysis of disordered systems. In particular, one gains a deeper understanding of the ergodic properties of the underlying dynamics. In that sense these concepts are the basis of numerous theoretical concepts. In particular, the characterisation of different time scales, of frozen degrees of freedom, and of aging processes becomes feasible. Thus such concepts are the centre of interest of a deeper microscopic understanding of equilibrium and nonequilibrium thermodynamics.

The present monograph summaries the scientific content of a WE Heraeus summer school held at Chemnitz University of Technology in August 2002. We owe the WE Heraeus foundation our deepest gratitude, as without the continuous and intense support from the foundation the whole project would not have been feasible. Whereas the scientific merits are due to our colleagues who have written the individual chapters, any remaining deficiencies must be attributed to the editors.

Chemnitz / London,
January 2004

*Günter Radons*
*Peter Häussler*
*Wolfram Just*

# Contents

Part I

# Pattern Formation and Growth Phenomena

# Introduction to Part I

Pattern formation is an ubiquitous non-equilibrium phenomenon which on one hand is simply fascinating, but on the other hand has many practical implications. It is one of the most prominent nonlinear collective dynamical effects, and therefore entails a long scientific tradition. Macroscopic patterns arising in fluids as a consequence of hydrodynamic instabilities are known for at least one century and are connected with prominent names such as Rayleigh-Benard or Taylor-Couette. In the meantime the importance of the pattern formation concept has been recognized in very diverse fields such as biology, laser physics, chemistry, and many others.

Quite recently pattern formation on a much smaller scale attracted the interest of the scientific community. Due to the development of advanced microscopy techniques such as the scanning tunnelling microscope, patterns that arise on a submicron scale during the epitaxial growth of solid surfaces, became observable. Their origin and understanding is the topic of the first contribution by Joachim Krug. He presents a phenomenological continuum theory, which provides a unified framework for various pattern forming mechanisms for surface growth. They include the Villain instability, mound and meandering instabilities, or patterns arising due to electromigration. Replacing the adsorption of atoms or molecules by the "deposition of vacancies" leads from growth to erosion phenomena which are caused e.g. by the bombardment of surfaces with high energy ions, and which can also be explained by such theories. The theoretical aspects include the wavelength selection problem and nonlinear evolution scenarios such as coarsening and chaotic bubbling.

The second contribution by Eckehard Schöll also deals with a very recent development, the nonlinear dynamics and pattern forming mechanisms in nanostructured semiconductors. The self-organized formation of current filaments, stationary or moving fronts, and other complex spatio-temporal or chaotic behavior are elucidated. Thereby the role of a negative differential conductivity is emphasized. Totally new aspects arise with the possibility of applying time-delayed feedback control, which is introduced and explored in this article. This opens the possibility to stabilize patterns and motions which are usually unstable and therefore not observable. The consequences of such an approach are discussed for resonant tunnelling diodes and for semiconductor superlattices.

The previous two articles dealt with pattern formation in selected applications of recent interest. In both cases insights are gained mostly within the framework of continuum theories. The latter constitute the central topic also in the contribution of Rudolf Friedrich. Now, however, no special application is aimed at, but consequences of symmetries of the system at hand are elaborated. The group theoretical results obtained can be applied to macroscopic or microscopic phenomena as long as a continuum description holds. It provides a characterization of non-equilibrium phase transitions or bifurcations through the classification of the order parameter equations which determine the patterns being formed. Finally the role of Goldstone modes for higher instabilities is explained and the experimentally relevant implications of fluctuations for dissipative solitons, such as filaments, are clarified.

The last article in this part by Peter Häussler deals with structure formation on a microscopic level. It is concerned with self-organization principles which aim at explaining the short-range ordered structure in disordered solids such as alloys. The paper advocates spherical periodic order and resonant behavior between electronic and structural degrees of freedom as the organizing principle, which can explain many properties in different families of alloys. Consequences e.g. for the density of states, transport properties, and dynamic excitations are discussed. In many respects this contribution leads over to Part II, which is exclusively devoted to the dynamical properties of disordered systems.

Part II

# Dynamics of Disordered Systems

# Introduction to Part II

Disordered systems provide a continuing challenge for the scientific community. Such systems often exhibit anomalous long-time behavior, which may be hard to access experimentally and difficult to explain theoretically. Such a behavior is found, for example, at the glass transition from undercooled liquids to the amorphous solid state. In addition non-equilibrium phenomena such as aging and anomalous transport are typically found. The difficulty of this subject lies in the appearance of non-trivial cooperative phenomena, which may result from competing interactions, e.g. between coupled spins, or from complex dynamic modifications of structural properties as in glasses.

The first two contributions to this part review experimental aspects of the dynamics of disordered systems. The first one by Reiner Zorn and Ulrich Buchenau is devoted to the structural glass transition. They show that suspensions of colloidal particles, such as polymer spheres, provide ideal systems for investigations of the glass transition. They are ideal in two respects: firstly experiments can be simply conducted with light (dynamic light scattering, DLS) instead of neutrons, which have to be used to probe the dynamics on a molecular scale. And secondly, these systems appear to be perfectly described by mode-coupling theory (MCT). The latter is only briefly sketched here, but treated in detail in the contribution by Rolf Schilling. In addition the differences between colloidal suspensions and undercooled liquids and dynamical mechanical measurements at the glass transition are discussed.

Jens-Boie Suck in his article reviews the experimental aspects of neutron inelastic scattering (NIS) for investigations of the dynamics of disordered systems. After introducing the density correlation functions measured in these experiments, the specific aspects for the observation of collective excitations in amorphous solids and in fluids are presented. In particular the successful applications of this technique e.g. to liquid metals or metallic glasses are reviewed, but also its current limitations are discussed.

In the following two contributions theories for the glass transition and glassy dynamics are presented and reviewed. The paper of Rolf Schilling deals mostly with the structural glass transition, where, in contrast to the spin-glass transition, static disorder is self-generated as result of this transition. After a concise review of several phenomenological theories, the currently most prominent microscopic theory for the structural glass transition, the mode-coupling theory, is derived and its consequences are explained in detail. In the framework of MCT the glass transition appears as a dynamical phase transition as it is identified as an ergodic-to-nonergodic transition, which can be measured via the intermediate scattering function obtainable from DLS or NIS experiments. Subsequently another prominent microscopic theory, replica theory, is treated in detail and it is shown that the latter yields a static glass transition. The possibly complimentary nature of these theories is discussed.

Heinz Horner's contribution is mainly concerned with mean-field like models and theories for glassy dynamics and in addition puts more emphasis on non-equilibrium phenomena such as aging. He first gives an overview over disordered systems, for which glassy dynamics is relevant. These range from spin-glasses to structural glasses, but also from systems such as neural networks to combinatorial optimization problems. The main part is devoted to dynamic mean-field theory and its consequences for the dynamics of the p-spin interaction spin-glass. It is shown how this theory yields results analogous to MCT for temperatures above the glass transition. In addition the non-equilibrium dynamics in the regime below the glass transition temperature is captured, which allows e.g. the description of aging phenomena. A comparison with replica theory is made and common as well as distinct features are worked out. Finally it is shown how properties of cage relaxation in supercooled liquids can be captured in such spin-glass models by including the possibility of relaxing bonds.

The last paper in this part by Ted Janssen presents models and theories for the nonlinear dynamics of aperiodic crystals. The term 'aperiodic'refers to a state, which is intermediate between random and periodic structures. After providing some typical examples in the class of quasi-crystals and from incommensurate structures, their description in superspace is explained. The main part of the contribution deals with models for these structures, such as the generalized Frenkel-Kontorova model or the Double Chain Model, and the dynamical collective excitations that can occur. Of special interest are the stability properties of phasons, characteristic low frequency excitations in incommensurate structures, and of solitons, both of which may decay, e.g. due to anharmonic interactions with phonons. Nonlinearity plays a two-fold role insofar as it is responsible for the occurrence of so-called phason gaps, but also because the ground state of these model systems is related to low-dimensional nonlinear mappings. The advancement in the understanding of the latter provides deeper insights into the ground state properties of these structural models and vice versa.

# Part III

# Complex and Chaotic Behaviour

# Introduction to Part III

The nonlinear dynamics of interacting particles or systems is generally accepted as one of the corner-stones of equilibrium and non-equilibrium statistical mechanics. Although it is clear that unstable and chaotic behaviour on a microscopic scale is of fundamental importance in this field, there exists, in contrast to systems with few degrees of freedom, still a considerable lack of understanding. One reason is the appearance of qualitatively new, collective effects in the thermodynamic limit. Also in the more general context of interacting entities in biological, social, or economic systems, the understanding of emerging complex behaviour is still in its infancy. This reasoning explains why in the following contributions the interplay between Nonlinear Dynamics and Statistical Physics plays a major role.

The first contribution by Günter Radons is naturally connected to the previous part on disordered systems: It is concerned with nonlinear dynamics in the presence of quenched disorder. He first points out that even quite simple disordered dynamical systems with one chaotic degree of freedom can naturally give rise to non-equilibrium phenomena such as aging, anomalous transport, and the dynamical phase transitions usually observed in complex many-particle systems. The second half of the paper is devoted to large ensembles of interacting limit cycle oscillators, i.e. prototype elements describing periodically active subsystems such as neurons or cells. The randomness lies in the individual frequencies and possibly in the interaction terms. In the first case, which is known as Kuramoto model a famous collective effect, the synchronization transition is explained in detail and the corresponding Lyapunov spectrum is calculated analytically. For random interactions the non-equilibrium counterpart of a spin-glass transition is identified and its manifestation in the Lyapunov spectrum is investigated.

The Lyapunov spectra of many particle systems is also one of the central objects of interest in the second paper by Harald Posch and Christina Forster. The corresponding perturbation dynamics is introduced and investigated in detail for fluids in equilibrium and in stationary non-equilibrium situations. As models, hard sphere gases or many-particle systems with soft interaction potential are investigated by molecular dynamics methods. The authors present an interesting recent finding in the equilibrium situation, namely the occurrence of steps in the Lyapunov spectrum of hard sphere systems and the observation of wave-like structures in the associated Lyapunov vectors. These hydrodynamic Lyapunov modes provide an example of a surprising new collective effect in chaotic many particle systems. The second part of this contribution is devoted to the non-equilibrium molecular dynamics (NEMD) of such systems. In systems which are to be kept in a stationary state far from equilibrium one needs thermostats to get rid of the excess heat produced by the external mechanical or thermal forces. The authors discuss how such states are reflected in the Lyapunov spectra of these systems, and how the latter are connected to macroscopic transport coefficients. Some aspects of this approach, e.g. the appearance of multifractal phase space structures, are subject of an ongoing scientific discussion to which the present article is a profound contribution.

The link between Nonlinear Dynamics and Equilibrium Statistical Physics is also the subject of the contribution of Wolfram Just. Here, however, the connection is more on a formal level where the analogy between time series of symbols and, in the simplest case, spin-chains is exploited. This subject is usually referred to as symbolic dynamics or the thermodynamic formalism. In this article first an elementary introduction to this topic is given. The concepts of Markov partitions and Markov maps are explained with the aid of simple models, which are helpful also for the understanding of the more complicated dynamics of the maps introduced in the first contribution by Günter Radons in this part. Subsequently these concepts are extended to deal with questions of space-time chaos, a subject of great current interest. For this purpose so-called coupled map lattices are introduced, which are constructed such that the Markov property is preserved. The time series generated by such an infinite one-dimensional array of coupled maps can be understood in terms of the equilibrium properties of the 2-dimensional Ising model, which is known to display a phase transition. This implies the occurrence of a new kind of dynamical phase transition for coupled maps, a macroscopic collective effect due to the extended nature of the system. This phenomenon is discussed as a special case of similar effects in more general coupled maps, where due to the typically greater complexity of the symbolic dynamics also transitions to spin-glass phases etc. can be expected.

The last contribution to this part by Heinz Georg Schuster introduces a quite different and novel aspect. The concept of adaptivity opens a whole realm of applications of nonlinear dynamics for instance in the biological sciences and in economy. As an example, agents modify their actions in response to reactions of other agents with the goal of optimising some environmental influence on them. Such problems define coupled nonlinear systems, which can be investigated with the tools of Nonlinear Dynamics. The approach is explained with two examples. The first is a self-recognition problem consisting in the determination of the status of a member in a hierarchy, and the second is the solution of the minority game with the aid of adaptive probabilistic Boolean functions.