

DYNAMICAL SYSTEMS METHOD FOR SOLVING OPERATOR EQUATIONS

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Abstract

Consider an operator equation $F(u)=0$ in a Hilbert space H and assume that this equation is solvable. Let us call the problem of solving this equation ill-posed if the operator $F'(u)$ is not boundedly invertible, and well-posed otherwise. A general method, Dynamical Systems Method (DSM), for solving linear and nonlinear ill-posed problems in H is presented. This method consists of the construction of a dynamical system, that is, a Cauchy problem, which has the following properties:

- 1) it has a global solution,
- 2) this solution tends to a limit as time tends to infinity,
- 3) the limit solves the original linear or non-linear problem.

The DSM is justified for

- a) an arbitrary linear solvable equations with bounded operator,
- b) for well-posed nonlinear equations with twice Fréchet differentiable operator F ,
- c) for ill-posed nonlinear equations with monotone operators,
- d) for ill-posed nonlinear equations with non-monotone operators from a wide class of operators,
- e) for operators such that $A := F'(u)$ satisfies the spectral assumption: $\|(A + sI)^{-1}\| \leq c/s$, where $c > 0$ is a constant, and $s \in (0, s_0)$, $s_0 > 0$ is a fixed number, arbitrarily small, c does not depend on s and u ,
- f) for some monotone operators which are not Fréchet differentiable, and
- g) for some unbounded, closed, densely defined F .

In Newton-type schemes the main difficulty is to invert the derivative of the operator. A novel scheme, based on the DSM, allows one to avoid this inversion.

A global convergence theorem is obtained for the regularized continuous analog of Newton's method for monotone operators. Global convergence means that convergence is established for an arbitrary initial approximation, not necessarily the one which is sufficiently close to the solution.

A general approach to constructing convergent iterative schemes for solving well-posed nonlinear operator equations is described and convergence theorems are obtained for such schemes.

Stopping rules for stable solution of ill-posed problems with noisy data are given.

References

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