Nicholas Watkins: *Linear fractional stable motion: a diffusion equation, apparent multifractality, and applications to space physics.*

Abstract:
In the 1960s Mandelbrot developed the use of fractals to describe how the shape of many aspects of the natural world departs from the Euclidean. In particular he proposed two kinds of fractal model to capture the way in which natural data is often persistent in time (his “Joseph effect”, common in hydrology and modelled by fractional Brownian motion) and/or prone to heavy tailed jumps (the “Noah effect”, typical of economic index time series and modelled by Lévy flights). Both effects are now accepted in proxies for auroral currents and and for the turbulent solar wind which is their ultimate energy source. Modelling them, however, has usually emphasised one of the Noah and Joseph parameters (the tail exponent $\mu$ and the temporal exponent $\beta$ respectively) at the other's expense.

In this talk I will first describe recent work [1] with Dan Credgington and co-workers at BAS and Warwick, in which we have applied a simple self-affine stable model- linear fractional stable motion (LFSM)-which unifies both effects to give insight into space physics data. I will argue that we have resolved some contradictions seen in earlier work, where purely Joseph or Noah descriptions had been sought.

Such hybrid Noah-Joseph “ambivalent” [2] behaviour is highly topical in physics. It is typically studied in the paradigm of the continuous time random walk (CTRW) rather than LFSM. Intriguingly the self-similarity exponent extracted from the CTRW differs from that seen in LFSM, being a ratio of $\mu$ and $\beta$ rather than an additive function. I will try to elucidate the physical differences between these two pictures with reference to a newly derived diffusion equation for LFSM.

I will touch on the use of an LFSM generator and simple analytic scaling arguments to study the problem of the area between a fractional Lévy curve and a threshold- directly related to the “burst size” measure introduced by Takalo and Consolini into space physics and studied by Freeman et al [3,4].

Finally I will discuss the how LFSM gives the appearance of multi-affine scaling without having an underlying cascade or multiplicative process (such as a turbulent cascade). The importance of this property for the interpretation of natural time series will be discussed.

[1] Watkins et al, Space Sci. Rev. 121, 271 (2005).
[2] Brockmann et al, Nature 439, 462 (2006).
[3] Freeman et al, Geophys. Res. Lett. 27, 1367 (2000).
[4] Freeman et al, Phys. Rev. E 62, 8794 (2000).