Problem 21

Consider the map \( f : [0, 1] \to [0, 1] \) defined by

\[
f(x) = \begin{cases} 
2(x + 1/6) & \text{if } 0 \leq x \leq 1/3 \\
-3(x - 2/3) & \text{if } 1/3 < x < 2/3 \\
x - 2/3 & \text{if } 2/3 \leq x \leq 1 
\end{cases}
\]

a) Sketch the graph of the map, determine a Markov partition, and state the corresponding topological transition matrix.

b) Is the map expanding? Is the map a piecewise linear Markov map? (state the reasons)

c) State all admissible periodic symbol sequences of period \( p = 2 \). Compute \( N_p \) - the number of periodic points of period \( p \) of the map \( f \) - for \( p = 1, 2, 3 \). Compute the asymptotic behaviour of \( N_p \) for large \( p \).

d) State the transfer matrix of the map \( f \). Compute the invariant density and the Lyapunov exponent.

e) Consider the following piecewise constant functions

\[
g(x) = \begin{cases} 
1 & \text{if } 0 \leq x \leq 1/3 \\
1 & \text{if } 1/3 < x < 2/3 \\
0 & \text{if } 2/3 \leq x \leq 1 
\end{cases}, \quad h(x) = \begin{cases} 
0 & \text{if } 0 \leq x \leq 1/3 \\
1/2 & \text{if } 1/3 < x < 2/3 \\
-1/2 & \text{if } 2/3 \leq x \leq 1 
\end{cases}
\]

Compute the correlation function \( C_{gh}(n) \) for \( n > 0 \). Sketch the correlation function in a diagram.

Problem 22**

Show that the tent map \( T : [0, 1] \to [0, 1], T(x) = 1 - |2x - 1| \) is topological transitive. (Hint: use the so called cylinder sets \( I_{\sigma_0 \sigma_1 \ldots \sigma_n} \) which have been introduced in the proof of proposition 2.1, and use the fact that these sets for large \( n \) yield a sufficiently fine partition of \([0,1]\). Prove transitivity by evaluating \( T^k(I_{\sigma_0 \sigma_1 \ldots \sigma_n}) \).)