Problem 19

Consider two maps $f : I \to I$ and $g : I' \to I'$ which are conjugate via a smooth transformation $h : I \to I'$, i.e. $h(f(x)) = g(h(x))$. Let $\rho_f : I \to \mathbb{R}^+$ denote an invariant density of the map $f$, i.e. $\rho_f(x)$ is a solution of the Frobenius-Perron equation of the map $f$. Show that $\rho_g(y) = \rho_f(h^{-1}(y)) / |h'(h^{-1}(y))|$ yields a solution of the Frobenius-Perron equation of the map $g$.

Problem 20

Consider the logistic map $F_4 : [0, 1] \to [0, 1]$

$$F_4(x) = 4x(1-x).$$

a) State the Frobenius-Perron equation for the logistic map.

b) Use the (smooth) conjugacy between the tent map and the logistic map, the invariant density of the tent map, and the relation between invariant densities stated in problem 19, to compute the invariant density of the logistic map.

c) Confirm that your expression for the invariant density of the logistic map solves the Frobenius-Perron equation of part a).