

School of Mathematical Sciences

MTH3100 Essential Mathematical Skills

Wednesday 13 February 2013, 1400

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

To pass the exam, you need 12 correct answers.

Record each answer by filling in the corresponding box in the answer form.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner: R.W. Whitty

1. Determine the number of primes lying between 89 and 109, end-points included

- [a] [a] 6 [b] 8 [e] not in the list
[c] 7 [d] 5

2. Determine the greatest common divisor x of 100^3 and 880

- [d] [a] $10 \leq x < 25$ [b] $25 \leq x < 50$ [e] not in the list
[c] $50 \leq x < 75$ [d] $75 \leq x < 150$

3. Determine the fractional part of $\frac{2019}{11}$

- [b] [a] $\frac{9}{11}$ [b] $\frac{6}{11}$ [e] not in the list
[c] $\frac{8}{11}$ [d] $\frac{2}{11}$

4. Evaluate

$$\left[-\frac{16}{3} \times \left(2 + \frac{3}{5} \right) \times \left(-\frac{1}{2} \right)^3 - \left(\frac{1}{2} - \frac{1}{3} \right) \right] \times \frac{21}{3} \times \left(-\frac{2}{47} \right) \div \left(\frac{2}{3} - 2 \right)$$

- [d] [a] $\frac{7}{5}$ [b] $\frac{4}{47}$ [e] not in the list
[c] $\frac{11}{47}$ [d] $\frac{7}{20}$

5. Estimate

$$x = \frac{10^{-3} \times 999}{8001^2} \times 2^6$$

- [e] [a] $10^{-3} < x < 10^{-2}$ [b] $10^{-4} < x < 10^{-3}$ [e] not in the list
[c] $10^{-5} < x < 10^{-4}$ [d] $10^{-6} < x < 10^{-5}$

6. Simplify

$$\frac{(-1)^2}{(-x)^{10}} \left(\frac{(-1/y)^3}{-x^{-3}} \right)^3$$

- [a] [a] $\frac{1}{xy^9}$ [b] $-\frac{1}{xy^9}$ [e] not in the list
[c] $\frac{1}{x^{19}y^9}$ [d] $-\frac{1}{x^4y^6}$

7. Compute the quotient of the following division

$$(4y^5 - y^4 + y^2 + 1) \div (y^3 + y^2 - 3)$$

- [a] [a] $4y^2 - 5y + 5$ [b] $4y^2 - 3y + 3$ [e] not in the list
[c] $4y^2 - 3y + 5$ [d] $4y^2 - 5y - 3$

8. When $35a^5b + 21a^3b^2 - 10a^6 - 6a^4b$ is factored completely, one of these factors is

- [b] [a] $7b + 2a$ [b] $5a^2 + 3b$ [e] not in the list
[c] $-2a^2 + 7b$ [d] $7a^2 + 2b$

9. Simplify

$$\frac{2 - 14x}{1 - 14x + 49x^2} - \frac{4 - x^2}{7x - 1}$$

- [c] [a] $-\frac{x^2 + 6}{7x - 1}$ [b] $\frac{x^2 - 2}{1 - 7x}$ [e] not in the list
[c] $\frac{x^2 - 6}{7x - 1}$ [d] $-\frac{x^2 + 2}{1 - 7x}$

10. Simplify

$$\left(\frac{a}{3} - \frac{b}{9}\right) \left[\frac{1}{4} \left(3a - \frac{2}{3}b\right)^2 + \left(\frac{3}{2}a + \frac{1}{3}b\right)^2 + \left(3a - \frac{2}{3}b\right) \left(\frac{3}{2}a + \frac{1}{3}b\right) \right]$$

- [c] [a] $(3a + b)a^2$ [b] $3a^3 - \frac{1}{3}a^2b - \frac{2}{9}ab^2$ [e] not in the list
[c] $3a^3 - a^2b$ [d] $(3a - b) \left(a^2 + \frac{4}{9}b^2\right)$

11. Compute $f(-x/y^2)$, where

$$f(\beta) = \frac{1}{-\beta + \frac{1}{\beta^3 - 3}}$$

- [d] [a] $-\frac{y^2(x^3 + 3y^6)}{x^4 + 3xy^6 + y^8}$ [b] $\frac{y^2(x^3 - 3y^6)}{x^4 - 3xy^6 - y^8}$ [e] not in the list
[c] $\frac{y^2(x^3 + 3y^6)}{-x^4 + 3xy^6 - y^8}$ [d] $\frac{y^2(x^3 + 3y^6)}{x^4 + 3xy^6 - y^8}$

12. Simplify, eliminating radicals at denominator

$$\left(\sqrt{20} - \sqrt{\frac{49}{5}} \right) \frac{4}{3 + \sqrt{5}}$$

[a] [a] $-3 + \frac{9}{5}\sqrt{5}$

[b] $\frac{15 + 9\sqrt{5}}{5}$

[e] not in the list

[c] $17 + \frac{51}{5}\sqrt{5}$

[d] $\frac{-75 + 9\sqrt{5}}{5}$

13. Simplify, eliminating radicals at denominator (assuming all radicands positive)

$$\sqrt{a^{-1} - 4a^3} \sqrt{(1 + 2a^2)^{-1}}$$

[b] [a] $\frac{\sqrt{a} \sqrt{1 + 2a^2}}{a}$

[b] $\frac{\sqrt{a(1 - 2a^2)}}{a}$

[e] not in the list

[c] $\frac{\sqrt{a(1 - 4a^4)}}{1 + 2a^2}$

[d] $\frac{\sqrt{a(1 - 4a^2)}}{a}$

14. Solve

$$\left(k + \frac{1}{2} \right)^2 - (k - 1)^2 - 6k < 5k - 3$$

[a] [a] $k > \frac{9}{32}$

[b] $k > \frac{3}{16}$

[e] not in the list

[c] $k < -\frac{15}{32}$

[d] $k > \frac{17}{40}$

15. Find all solutions of

$$\frac{1 - x}{3\sqrt{x - 2}} = 1$$

[c] [a] $\frac{11 + 3\sqrt{5}}{2}$

[b] $\frac{11 - 3\sqrt{5}}{2}$

[e] not in the list

[c] no solution

[d] $\frac{7 \pm 3\sqrt{3}}{2}$

END OF EXAMINATION PAPER