Number Systems and Fractals, Gary Michalek

The talk will explain a nice connection between the two fields of algebra (specifically modular arithmetic) and topology (specifically fractals). The basic idea is this: you can write any positive integer in base 3 using the symbols $A = \{0, 1, 2\}$, and therefore any integer can be written using the symbols $A - A = \{0, 1, 2, -1, -2\}$. But what happens if you change the coefficients of powers of three to other numbers, e.g. $A = \{0, 7, 17\}$. Can every integer be expressed as sums of powers of 3 with coefficients in $A - A = \{0, 7, -7, 17, -17, 10, -10\}$? (The answer is yes). I proved a theorem that shows which sets of coefficients work (are number systems) and which don't, both in base 3 and later for any integer base N. There is a conjectured theorem for the more complex case of Gaussian integers, base $1 + 4i$, for example. This is the algebra side of the talk. The set A as described above also corresponds to a fractal (if you use A for coefficients of powers of $1/3$ or $1/N$ in general, where N is a real or complex integer). The fractal is the fixed point of an iterated function system. Whether or not A is a number system is reflected in a nice topological property of the fractal that A creates (there are pictures!). This might be interesting in general to mathematicians but also for teachers of mathematics who would like examples for their courses in algebra or topology.