Network Valuation in Financial Systems

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CoSyDy - July 6, Queen Mary University of London

Research objectives

Question: *What is the net value of a financial institution in a network?*

Contribution: *We develop a new model that takes into account at the same time* ***interdependencies*** *(as in Furfine 1999, Eisenberg and Noe 2001) and* ***uncertainty*** *(as in Merton 1974)*

Relevance: *Network valuation is crucial for assessing systemic risk in interconnected systems, e.g. stress-tests and contagion processes,* ***but also for day-to-day pricing, i.e. valuation of claims***

Interdependent asset valuation under uncertainty

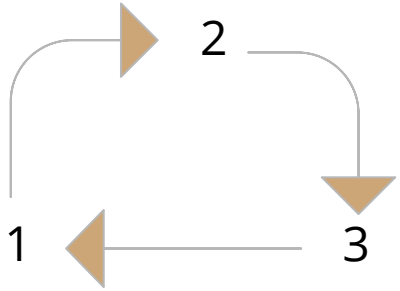
What is value of my intangible assets today (t)?

1) **Time Dimension:** An asset can be a contract to make a transaction at time T>t. Future implies uncertainty
2) **Space Dimension:** Asset value may depend on counterparties' asset values

1 → 2

3 → 1

2 → 3



System of non-linear equations with cyclical dependence → no guarantee of unicity nor existence of solutions

Selected relevant literature

- Merton 1974 *On the pricing of corporate debt: the risk structure of interest rates*
- Eisenberg and Noe 2001 *Systemic risk in financial systems*
- Battiston, Puliga, Kaushik, Tasca, and Caldarelli 2012 *DebtRank: Too Central to Fail? Financial Networks, the FED and Systemic Risk*
- Rogers and Veraart 2013 *Failure and rescue in an interbank network*
- Glasserman and Young 2014 *How likely is contagion in financial networks?*
- Bardoscia, Battiston, Caccioli, and Caldarelli 2015 *DebtRank: A microscopic foundation for shock propagation*
- Barucca, Bardoscia, Caccioli, D'Errico, Visentin, Caldarelli, Battiston *2016 Network Valuation in Financial Systems*

Time structure of asset valuation

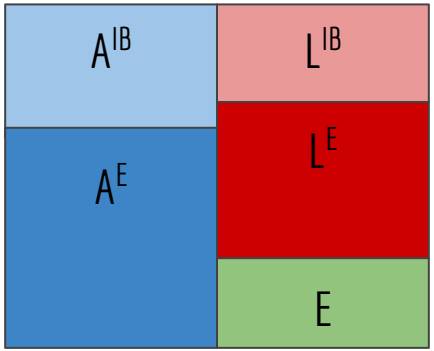
- Time 0: all liabilities are issued (with same maturity T)
- Time t - τ: a shock occurs (e.g. a steep change in the external assets)
- Time t: asset valuation is performed
- Time T: clearing procedure at maturity

Merton Model (τ =0)

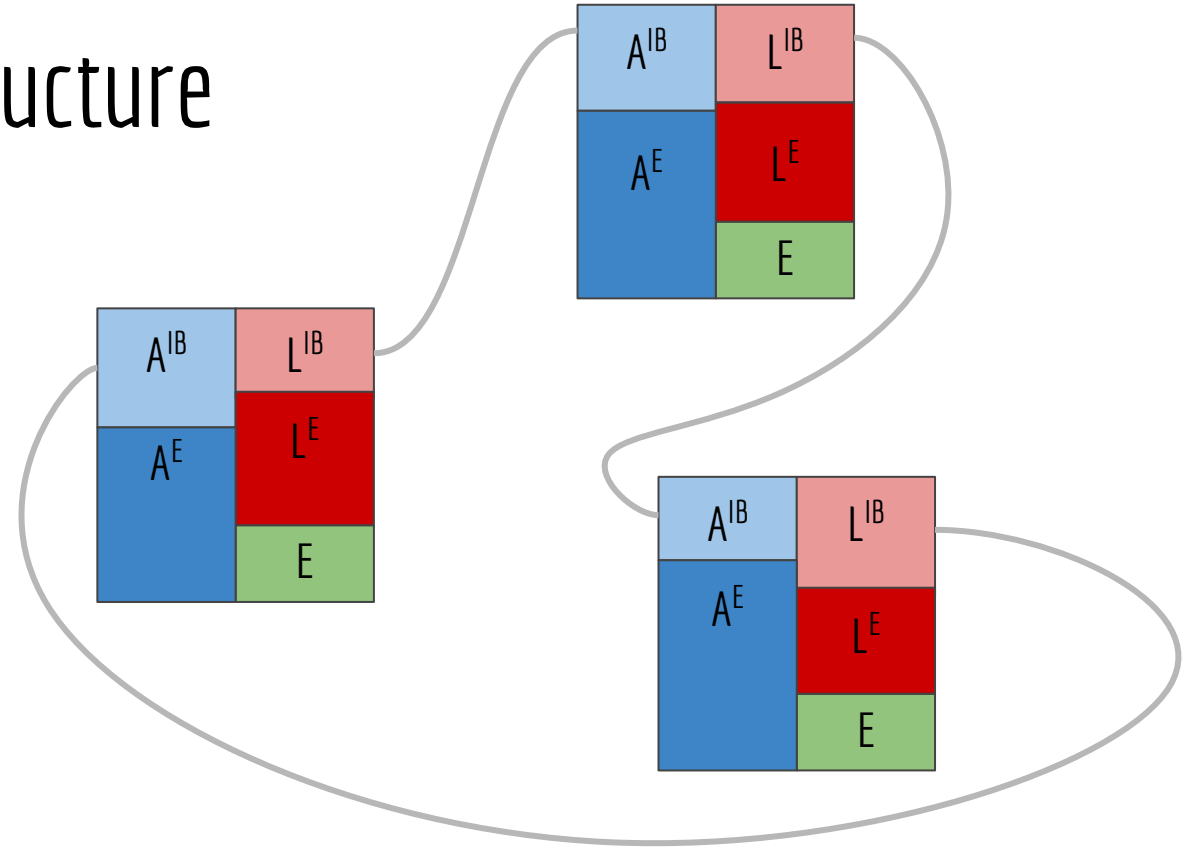
Eisenberg and Noe (τ =0, t=T)

NEVA

Interbank market structure



L^{IB}= INTERBANK LIABILITIES
A^{IB}= INTERBANK ASSETS
L^{E} = EXTERNAL LIABILITIES
A^{E} = EXTERNAL ASSETS
E = EQUITY



The market value of interbank assets depends on the interbank network

Network Valuation (NEVA)

$$\tilde{E}_{it} = \mathbb{V}_i^{(e)}(A_{it}^{(e)}|\tilde{E}_{it}) - L_i^{(e)} + \sum_j \mathbb{V}(A_{ij}|\tilde{E}_{jt},\ldots) - \sum_j L_{ij}$$

$$\frac{\mathbb{V}(A_{ij}|\tilde{E}_{jt},\ldots)}{A_{ij}} = \mathbb{E}[\mathbb{1}_{\tilde{E}_{jT}>0} + R(\frac{\tilde{E}_{jT}+\bar{p}_j}{\bar{p}_j})^+ \mathbb{1}_{\tilde{E}_{jT}\leq 0}]_{\Delta A_j^{(e)}}$$

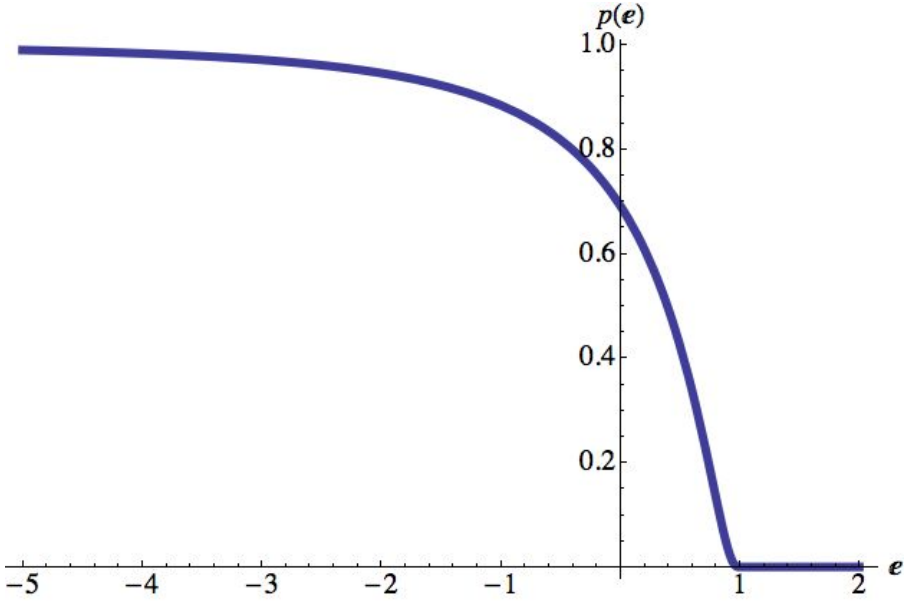
Proposition *A single step of the Picard algorithm associated to NEVA corresponds to a optimal pricing performed by each bank locally*

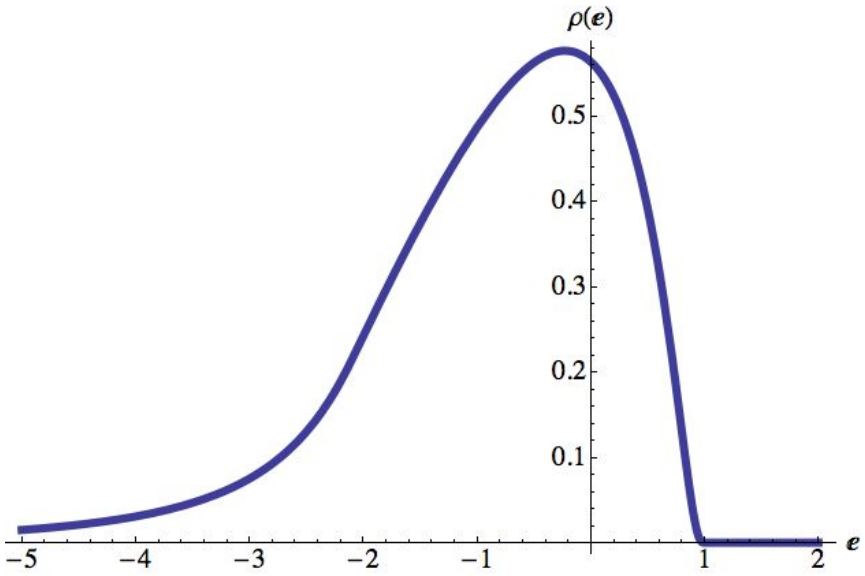
Summary table of valuation models

Model	***Valuation Time***	***Network Propagation***	***Default losses***	***Endogenous Recovery***
Merton	Ex-ante	None	None	None
Eisenberg Noe	Ex-post	Local	None	Full
Rogers Veraart	Ex-post	Local	Present	Full
Linear DebtRank	Ex-ante	Local	Present	None
Fischer Model	Ex-ante	Global	None	Full
NEVA	Ex-ante	Local	Present	Full

Endogenous valuation

$$\frac{\mathbb{V}(A_{ij}|\tilde{E}_{jt},\ldots)}{A_{ij}} = (1 - p^D(\tilde{E}_{jt})) + R\rho^D(\tilde{E}_{jt})$$





Results

Proposition NEVA converges to Eisenberg and Noe clearing procedure when the maturity goes to zero and the exogenous recovery R=1*

Proof Sketch *As uncertainty decreases the expected value is given by the most probable value that corresponds exactly to Eisenberg and Noe valuation when maturity goes to zero.*

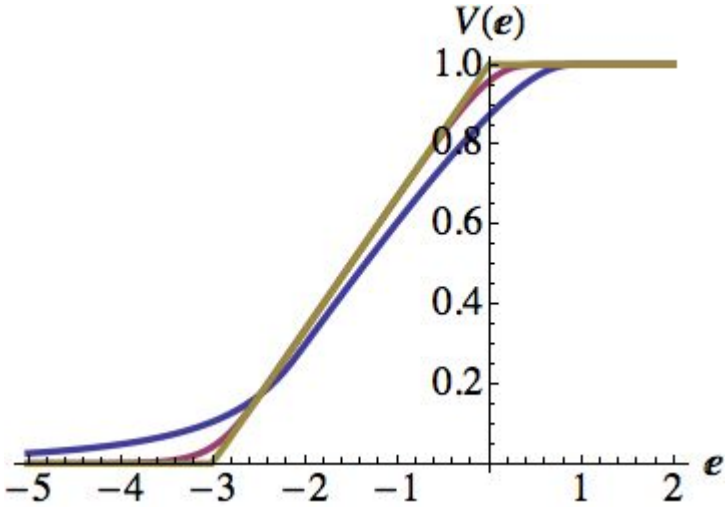
**Linear Threshold Model is recovered for R=0*

Results

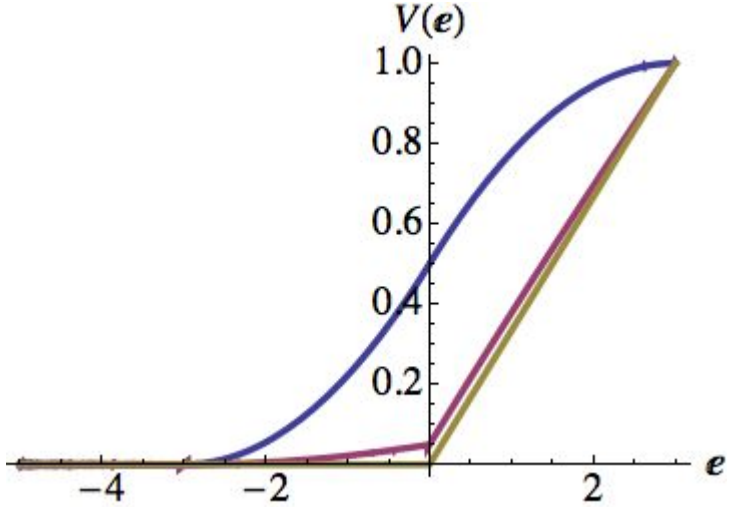
Proposition NEVA converges to Linear Debt Rank in the case of zero recovery and uniform distribution of shocks.

Proof Sketch *In the case of uniform distribution of shocks the probability of default given by the expected value of the default indicator function becomes linear while the endogenous recovery term is zero being multiplied by a zero exogenous recovery rate.*

A closer look at valuation functions



Convergence to Eisenberg and Noe valuation



Convergence to linear DebtRank valuation

Results

Let us define the iterative map $\mathbf{E}^{n+1} = \phi(\mathbf{E}^{n})$ with the initial condition $\mathbf{E}^{0} = \mathbf{M}$. Where **M** is the maximum possible equity value corresponding to a face-value asset valuation.

Theorem The sequence $\mathbf{E}^{n}$ converges to the optimal solution $\mathbf{E}^{*}$

Proof Sketch *Convergence relies on boundedness, monotonicity, and continuity from above of the map. If the map then converged to a solution lower than* $\mathbf{E}^{*}$ *there would be a contradiction with the order-preserving property of the Endogenous Debt Rank valuation function.*

Results

Theorem NEVA always admits a solution $\mathbf{E}^*$ that is the maximum of a complete lattice.

Proof Sketch *Based on Knaster-Tarski theorem. We just need to show that the equity space is bounded and that the valuation function is order-preserving.*

Conclusions

- We developed a **novel valuation model** that allows to carry out an **ex-ante** valuation of the claims in a network context in the presence of **uncertainty** deriving from **shocks** on the **external assets** of banks while at the same time providing an **endogenous** and **consistent recovery rate**.
- The new model encompasses both the **ex-post** approaches, **Furfine** (Linear Threshold Model), **Eisenberg and Noe**, and **Rogers and Veraart**, and the **ex-ante** approaches, **Merton** and **DebtRank**, in the sense that **each of these models can be recovered** with the appropriate parameter set.
- We characterize the **existence** and **uniqueness** of the **optimal** solution to the valuation problem and **provide an algorithm** to find it.

Perspectives

- Is network valuation a general feature of economic and social systems?
- When are local valuation processes as good as global valuation processes? Is valuation always decentralizable?
- Can we quantify the efficiency of a local valuation process?
- Does valuation play a role in network formation?

Thanks for the attention!