

MTH5105 Differential and Integral Analysis
MID-TERM TEST

Date: 26-02-2010 Time: 12:10–12:50

Complete the following information:

Name	
Student Number (9 digit code)	

The test has THREE questions. You should attempt ALL questions. Write your calculations and answers in the space provided. Cross out any work you do not wish to be marked.

Question	Marks
1	
2	
3	
Total Marks	

Nothing on this page will be marked!

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Question 1.

- (a) State the formula for the Taylor polynomial $T_{n,a}$ of degree n of a function f at a , and state the Lagrange form of the remainder term R_n . [10 marks]

Let $f(x) = 1/\sqrt{1+x}$.

- (b) Determine the Taylor polynomials $T_{2,0}$ and $T_{3,0}$ of degree 2 and 3, respectively, for f at $a = 0$. [15 marks]
- (c) Using the Lagrange form of the remainder term, or otherwise, show that

$$T_{3,0}(x) < f(x) < T_{2,0}(x) \quad \text{for all } x > 0.$$

[10 marks]

Answer 1.

Answer 1. (*Continue*)

Question 2.

(a) Give the definition of $f : \mathbb{R} \rightarrow \mathbb{R}$ being differentiable at a point $a \in \mathbb{R}$. [10 marks]

(b) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfy

(i) $f(x) = f(x - y)f(y)$ for all $x, y \in \mathbb{R}$, and

(ii) $f(x) - 1 = xg(x)$ with $\lim_{x \rightarrow 0} g(x) = 1$.

Show that f is differentiable and that $f'(a) = f(a)$ for all $a \in \mathbb{R}$. [20 marks]

Answer 2.

Answer 2. (*Continue*)

Question 3.

(a) State Rolle's Theorem.

[15 marks]

(b) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be twice differentiable with

$$f(0) = f(1) = f(2) = 0 .$$

Show that there exists a $c \in (0, 2)$ such that $f''(c) = 0$.

[20 marks]

Answer 3.

Answer 3. (*Continue*)