MTH5105 Differential and Integral Analysis
MID-TERM TEST
Date: 20-02-2009 *Time:* 2:10–2:50

Complete the following information:

Name	
Student Number (9 digit code)	

The test has THREE questions. You should attempt ALL questions. Write your calculations and answers in the space provided. Cross out any work you do not wish to be marked.

Question	Marks
1	
2	
3	
Total Marks	

Nothing on this page will be marked!

Question 1.

Let $f(x) = 1/x$.

(a) Determine the Taylor polynomials $T_{3,1}$ and $T_{4,1}$ of degree 3 and 4 at $a = 1$ for f.

[*15 marks*]

(b) Using the Lagrange form of the remainder, or otherwise, show that

$$T_{3,1}(x) < f(x) < T_{4,1}(x) \quad \text{for all } x > 1 \,.$$

[*15 marks*]

Answer 1.

Answer 1. (*Continue*)

Question 2.

(a) Give the definition of $f : \mathbb{R} \rightarrow \mathbb{R}$ being differentiable at a point $a \in \mathbb{R}$.

[*10 marks*]

(b) Using the definition, determine whether or not

$$f(x) = \begin{cases} \dfrac{x}{1 + \exp(1/x)} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is differentiable at $x = 0$. (For this you may wish to consider the left and right derivatives of $f(x)$ at $x = 0$.) Find $f'(0)$, if it exists. [*20 marks*]

Answer 2.

Answer 2. (*Continue*)

Question 3.

(a) State the Mean Value Theorem. [*15 marks*]

(b) Show that for all $x, y \in \mathbb{R}$

$$|\sin(y) - \sin(x)| \le |y - x| .$$

[*25 marks*]

You may assume standard properties of trigonometric functions.

Answer 3.

Answer 3. (*Continue*)