MTH5105 Differential and Integral Analysis 2010-2011

Exercises 1

There are two sections. Answers to questions in Section 1 will be used for feedback. Questions in Section 2 are voluntary but highly recommended.

1 Exercises for Feedback

1) Using the definition of the derivative of a function, investigate for which values of x each of the following two functions is differentiable, and find the derivatives, if they exist.

   (a) $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto (x+1)|x|$,

   (a) $g : \mathbb{R} \to \mathbb{R}$, $x \mapsto (x-1)|x-1|$.

2 Extra Exercises

2) Prove that the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 \sin(1/x^2) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

   is differentiable at zero and find $f'(0)$.

   Find $f'(x)$ for $x \neq 0$ assuming that $\sin' = \cos$.

   Give a rough sketch of the curve $f'(x)$ for small x and mark $f'(0)$ clearly on your sketch.

3) Let $f : [-1,1] \to \mathbb{R}$ be continuous on $[-1,1]$, differentiable at zero and $f(0) = 0$. Show that the function

$$g(x) = \begin{cases} f(x)/x & x \neq 0 \\ f'(0) & x = 0 \end{cases}$$

   is continuous at zero.

   Is g continuous for $x \neq 0$?

   Deduce that there is some number M such that

$$f(x)/x \leq M \quad \text{for all} \quad x \in [-1,1] \setminus \{0\} \, .$$

4) Give an example of a function that is differentiable on (a,b) but that cannot be made differentiable on $[a,b]$ by any definition of $f(a)$ or $f(b)$. Can you give an example where f is bounded?

The deadline is 5.00pm (strict) on Monday 24th January. Please hand in your coursework to the orange coursework box on the second floor. Coursework will be returned during the exercise class immediately following the deadline.

Thomas Prellberg, January 2011