Queen Mary
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Examiner: Dr T Prellberg

# MTH5105 Differential and Integral Analysis
## MID-TERM TEST
*Date:* 25 Feb 2011 *Time:* 12:10–12:50

## Complete the following information:

| | |
|---|---|
| **Name** | |
| **Student Number (9 digit code)** | |

The test has THREE questions. You should attempt ALL questions. Write your calculations and answers in the space provided. Cross out any work you do not wish to be marked.

| Question | Marks |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| Total Marks | |

**Nothing on this page will be marked!**

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**Question 1.**

(a) State the formula for the Taylor polynomial $T_{n,a}$ of degree $n$ of a function $f$ at $a$. [*10 marks*]

Let $g(x) = -\log(1-x)$.

(b) Prove by mathematical induction that $g^{(n)}(x) = \dfrac{(n-1)!}{(1-x)^n}$ for $n \in \mathbb{N}$. [*10 marks*]

Hence compute the Taylor polynomial $T_{4,0}$ of degree $4$ of $g$ at $a = 0$. [*10 marks*]

Let $h(x) = \log(1 + x + x^2)$.

(c) By factorising $1 - x^3$, or otherwise, determine the Taylor polynomial $T_{4,0}$ of degree $4$ of $h$ at $a = 0$. [*10 marks*]

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**Answer 1.**

**Answer 1.** (*Continue*)

**Question 2.**

(a) Give the definition of $f : \mathcal{D} \to \mathbb{R}$ being differentiable at a point $a \in \mathcal{D}$. [*10 marks*]

(b) Using this definition, show that $g(x) = 1/x$ is differentiable at $a = 1$ and find its derivative. [*10 marks*]

(c) Suppose that the function $f$ is continuous at $0$. Show that the function $h$ defined by

$$h(x) = xf(x)$$

is differentiable at zero and find its derivative. [*10 marks*]

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**Answer 2.**

**Answer 2.** (*Continue*)

**Question 3.**

(a) State the Mean Value Theorem. [*15 marks*]

(b) Let $g$ be differentiable on $[0, 1]$ with

$$g'(x) = 0$$

for all $x \in [0, 1]$, and let $g(0) = 1$. Using the Mean Value Theorem, or otherwise, prove that

$$g(x) = 1$$

for all $x \in [0, 1]$. [*15 marks*]

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**Answer 3.**

**Answer 3.** (*Continue*)