

# MTH5105 Differential and Integral Analysis 2008-2009

## Midterm Test

Problem 1: Let  $f(x) = 1/x$ .

- (a) Determine the Taylor polynomials  $T_{3,1}$  and  $T_{4,1}$  of degree 3 and 4 at  $a = 1$  for  $f$ .

[15 marks]

- (b) Using the Lagrange form of the remainder, or otherwise, show that

$$T_{3,1}(x) < f(x) < T_{4,1}(x) \quad \text{for all } x > 1 .$$

[15 marks]

Problem 2: (a) Give the definition of  $f : \mathbb{R} \rightarrow \mathbb{R}$  being differentiable at a point  $a \in \mathbb{R}$ .

[10 marks]

- (b) Using the definition, determine whether or not

$$f(x) = \begin{cases} \frac{x}{1 + \exp(1/x)} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is differentiable at  $x = 0$ . (For this you may wish to consider the left and right derivatives of  $f(x)$  at  $x = 0$ .) Find  $f'(0)$ , if it exists. [20 marks]

Problem 3: (a) State the Mean Value Theorem. [15 marks]

- (b) Show that for all  $x, y \in \mathbb{R}$

$$|\sin(y) - \sin(x)| \leq |y - x| .$$

[25 marks]

You may assume standard properties of trigonometric functions.