1. Consider the half-replicate fractional factorial design for three factors, each at two levels, plus two centre points:

\[
\begin{array}{ccc}
X_1 & X_2 & X_3 \\
-1 & -1 & -1 \\
-1 & 1 & 1 \\
1 & -1 & 1 \\
1 & 1 & -1 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{array}
\]

Find a 95% confidence interval for \( \beta_1 + \beta_{23} \) if \( \hat{\beta}_1 + \hat{\beta}_{23} = 10.03 \) and \( s^2 = 5.34 \). How would the result be interpreted?

2. Consider the following data:

\[
\begin{array}{cccc}
Y & X_1 & X_2 & X_3 \\
10 & -1 & -1 & -1.0001 \\
12 & -1 & 1 & -0.9999 \\
20 & 1 & -1 & 1.0000 \\
22 & 1 & 1 & 1.0000 \\
16 & 0 & 0 & 0.0000 \\
18 & 0 & 0 & 0.0000 \\
\end{array}
\]

(a) Test the hypothesis \( \beta_1 = \beta_2 = 0 \) against a two-sided alternative.
(b) Test the hypothesis \( \beta_1 = \beta_3 = 0 \) against a two-sided alternative.
(c) Calculate the leverage of each observation.
(d) Assuming that the experiment was completely randomized, carry out a permutation test of $\beta_1 = 0$ against a two-sided alternative.

(e) Comment on the results.

3. For the general second order polynomial regression model, with $q$ explanatory variables, find the location of the stationary point as a function of the parameters. (Hint: rewrite the model using the vector $b' = [\beta_1 \cdots \beta_q]$ and the matrix

$$
B = \begin{bmatrix}
\beta_{11} & \frac{1}{2}\beta_{12} & \cdots & \frac{1}{2}\beta_{1q} \\
\frac{1}{2}\beta_{12} & \beta_{22} & \cdots & \vdots \\
\vdots & \ddots & \ddots & \frac{1}{2}\beta_{(q-1)q} \\
\frac{1}{2}\beta_{1q} & \cdots & \frac{1}{2}\beta_{(q-1)q} & \beta_{qq}
\end{bmatrix}
$$

and then use vector differentiation.)