MTH6111 Complex Analysis 2010-11

Exercises 5

Attempt all questions. Hand in your solutions to Questions 1, 3 and 5 for coursework credit and feedback. You have an extra week as 21-25 Feruary is READING WEEK. There will be no lectures in this course that week: you should spend time revising your notes and reading textbooks, in particular finding out more about the applications of Cauchy's Theorem and the Residue Theorem.

***1. [5 marks]** By applying Cauchy's Estimate for $f'(z)$ show that every entire function f which is bounded has zero derivative everywhere. Deduce that f is constant by a result from earlier in the course. (This gives anouther proof of Liouville's Theorem).

2. Let f be holomorphic on $D(a, R)$ and let $0 < r < R$.
(i) Prove that

$$f(a) = \frac{1}{2\pi} \int_0^{2\pi} f(a + re^{i\theta}) d\theta \quad \text{(Gauss's Mean Value Theorem)}$$

Deduce that either there exists $z \in C(a, r)$ with $|f(z)| > |f(a)|$, or $|f(z)| = |f(a)|$ for all $z \in C(a, r)$.
(ii) *(Harder)* Deduce from (i) that if f is non-constant then there does not exist a point $b \in D(a, R)$ such that $|f(z)| \leq |f(b)|$ for all $z \in D(a, R)$.
(HINT: Apply part (i) to a disc centred at b and contained in $D(a, R)$. Then apply Taylor's Theorem.)

***3. [5 marks]** Find the Laurent series for $f(z) = (z-1)^{-1}(z-2)^{-1}$ on each of the following annuli: $A_1 = \{z : 0 < |z| < 1\}$; $A_2 = \{z : 1 < |z| < 2\}$; $A_3 = \{z : 2 < |z| < \infty\}$.

4. Find the residue of $f(z) = (z^3 - z^5)^{-1}$ at each of its isolated singularities.

***5. [10 marks]** By applying the Residue Theorem, compute

$$\int_{C(0,2)} \frac{1}{(z-1)^2(z^2+1)} dz.$$

Deadline for handing in solutions to questions 1, 3 and 5: 12 noon on (exceptionally) Monday 28 February 2011. *(To me or to the yellow box on the second floor of the Maths building.)* SB 10/2/11