Extrasolar Planets and Astrophysical Discs

Revision Lecture 2011

The exam : 3 hours long.

Calculators are allowed !

Section A : 4 Compulsory questions - 50 Marks.

Section B : Answer 2 questions from 3 - 50 marks.

Important things to know :

(1). The Virial theorem - be able to derive the expression

$$\frac{d^2 I}{dt^2} = 4K + 2E_g + 6\int_V P\,dV \quad (1)$$

(neglecting the magnetic terms).

Know that the condition for collapse is $\frac{d^2 I}{dt^2} < 0$.

Use eqn (1) and the condition for collapse to make estimates for the critical mass for collapse.

(2). Estimate disc sizes from collapse of rotating molecular clouds : $R_{disc} = \frac{j^2}{GM}$.

③. Disc formation in close binary systems – be able to derive the Roche potential

$$\Phi(x,y) = \frac{-GM_1}{\sqrt{(x-x_1)^2+y^2}} - \frac{GM_2}{\sqrt{(x-x_2)^2+y^2}} - \frac{1}{2}\Omega^2(x^2+y^2).$$

Be able to sketch and label the Roche equipotentials.

Estimate size of ring of gas that forms around primary star due to mass transfer through L1 point – understand why accreting objects must be compact.

Be able to derive from first principles the expression

$$\frac{\dot{D}}{D} = \frac{2|\dot{M}_2|}{M_2}\left(1 - \frac{M_2}{M_1}\right)$$

where D is the binary separation.

Also be able to describe how system evolves depending on whether the accreting star M_1 is more or less massive than the donor star.

④. Angular momentum transfer mechanisms:
Viscosity; Magnetic fields + disc wind; Spiral shock waves.
Be able to describe those mechanisms.

(5) Be able to derive the diffusion eqn:

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}\left[\left(\frac{\partial j}{\partial R}\right)^{-1}\frac{\partial}{\partial R}\left(R^3 \nu \Sigma \frac{d\Omega}{dR}\right)\right] = 0. \qquad (2)$$

Be able to obtain eqn (2) in its Keplerian form

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}\left(R^{1/2}\nu\Sigma\right)\right]$$

and obtain the viscous evolution time scale

$$t = \frac{R^2}{3\nu}.$$

Be able to make numerical estimates for the value of α when adopting the "alpha" model for viscosity $\nu = \alpha c_s H = \alpha H^2 \Omega$, for a given viscous evolution time (e.g. outbursts in C.V. discs).

Be able to derive steady-state solution for eqn (2):

$$\nu\Sigma = \frac{\dot{M}_d}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right].$$

(6). Be able to derive the expression

$$T_{eff} = \left\{ \frac{3GM\dot{M}_d}{8\pi\sigma R^3} \left(1 - \left[\frac{R_*}{R}\right]^{1/2} \right) \right\}^{1/4}$$

Using the energy dissipation rate $\epsilon_D = \frac{9}{4} \nu \Sigma \Omega^2$.

and the radiative loss rate $F = \sigma T_{eff}^4$.

Make numerical temperature estimates for given disc parameters, and understand in which region of the electromagnetic spectrum a disc with a given T_{eff} will emit primarily.

Be able to derive Eddington limit for accretion onto compact objects $\dot{M}_{edd} = \frac{4\pi GM}{c\kappa}$. Understand physical origin of this limit.

(7). Boundary layers:

Be able to show that the total energy emission rate from a disc that is accretion matter from $R = \infty$ to $R = R_*$ is $L = \frac{GM\dot{M}_d}{2R_*}$.

Understand that this is <u>half</u> of the available energy, and be able to explain where the rest of this energy is stored.

Role of boundary layers in providing source of high energy photons.

(8). Extra Solar Planets

Be able to describe the main detection techniques, and the types of Planetary Systems that they can observe: radial velocity, transit, thermal infrared detection, microlensing, direct imaging. Be able to describe the pros & cons of each technique.

Given an example of radial velocity data, be able to estimate planet mass. Or, given planet mass and orbital configuration, be able to determine the required sensitivity for detecting the radial velocity of the central star: $M_p \sin i = V_{obs} M_* \sqrt{\frac{D}{GM_*}}$.

Given transit data (e.g. relative dimming of star during transit) be able to determine planet properties.

(9) Planet formation:

(i). Understand and be able to describe the basic properties of protoplanetary discs, and the minimum mass solar nebula model.

(ii). Be able to derive the expression

$$v \simeq \frac{8 a \rho_{gr} \Omega H}{3 \Sigma}$$

for the vertical terminal velocity of dust grains as they settle toward the midplane.

Be able to derive the time scale for grains to settle $\tau_s \simeq \frac{\Sigma}{16 a \rho_{gr}}$ orbits

(iii). Be able to estimate the mass and physical size of the dust grains / pebbles as they reach the midplane after settling : $M_{z=0} = \left(\frac{3}{4\pi \rho_{gr}}\right)^2 \left(\frac{\Sigma}{2 \rho_{gd}}\right)^3$

(iv).

Be able to derive the expression for the rate of planetesimal growth $\frac{dM_c}{dt} = \pi n m v R_c^2 \left(1 + \frac{2 G M_c}{R_c v^2}\right)$ (3) from first principles.

(v). Understand and be able to explain the concepts "runaway growth" and "orderly growth".

Be able to derive the timescale for runaway growth of a planetesimal from eqn (3).

$$t_G = 3 M_c^{-1/3} \left(\frac{4\pi \rho_{gr}}{3}\right)^{1/3} \frac{H_s^2 \Omega}{\pi G \Sigma_s}$$

(vi). Understand the two main competing models of giant planet formation: the core instability model and the gravitational instability model.
Be able to describe these models, and be able to make estimates about the conditions which must prevail for each model to operate.

(vii). Planetary Migration – be able to describe the different mechanisms that can cause planets to migrate: gas disc interaction; planet - planet scattering; planetesimal disc interaction. Understand type I and type II migration. Be aware of the migration rates associated with each of them.

(viii) Gap formation: Be able to derive the condition for gap formation

$$\boxed{\frac{M_p}{M_*} > \frac{27\pi\nu}{8R^2\Omega}}$$

Be able to make numerical estimates for the masses of planets that will open gaps, subject to suitable conditions in the disc.
Understand how the viscous evolution of the disc can drive type II migration.

(ix) Be able to describe the so-called Nice Model, in which the Solar System underwent a late-stage dynamical instability: late heavy bombardment, capture of trojan asteroids, eccentricities & inclinations of the giant planets, structure of the Kuiper belt.