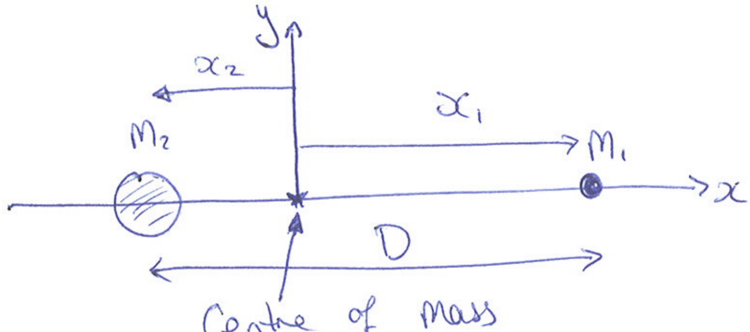
Mass Transfer in Close Binaries and Changes in Orbital Separation.

The process of transferring mass through Robe Lobe overflow in a close binary system will change its mass ratio $q = \frac{M_2}{M_1}$ where M_1 is the accreting compact object and M_2 is the mass of the donor star.

One consequence of this is that this leads to a change in the orbital separation, D, and period, P. This turn leads to a change in the Roche geometry, which may change the way in which the donor star fills its Roche lobe.



The angular velocity associated with the circular orbit of each star is Ω.

The total angular momentum of the system

$$J = (M_2 x_2^2 + M_1 x_1^2)\Omega \quad (1)$$

where $x_1 = \frac{D M_2}{(M_1 + M_2)}$ $\quad x_2 = -\frac{D M_1}{(M_1 + M_2)}$.

We assume that J is conserved during mass transfer between the donor star and the accreting object.

Substituting the expressions for x_1 and x_2 into ①.

$$J = \left(\frac{M_2 M_1^2 D^2}{(M_1+M_2)^2} + \frac{M_1 M_2^2 D^2}{(M_1+M_2)^2} \right) \Omega$$

where $$\Omega = \sqrt{\frac{G(M_1+M_2)}{D^3}}$$

$$\therefore J = \frac{D^2}{(M_1+M_2)^{\cancel{2}}} \left[\cancel{(M_1+M_2)} M_1 M_2 \right] \sqrt{\frac{G(M_1+M_2)}{D^3}}$$

$$\therefore J = M_1 M_2 \sqrt{\frac{GD}{(M_1+M_2)}} \qquad ②$$

We assume that all mass lost by M_2 $\left(\frac{dM_2}{dt} < 0\right)$ is gained by mass M_1 $\left(\frac{dM_1}{dt} > 0\right)$ such that $\frac{dM_2}{dt} = -\frac{dM_1}{dt}$.

[Note: $\frac{dM}{dt}$ is the mass accretion rate]

Differentiate ② w.r.t. time; denoting time derivatives with a dot

$$\frac{dJ}{dt} = \dot{J} = \dot{M}_1 M_2 \sqrt{\frac{GD}{(M_1+M_2)}} + \dot{M}_2 M_1 \sqrt{\frac{GD}{(M_1+M_2)}} + \frac{1 M_1 M_2}{2} \sqrt{\frac{G}{D (M_1+M_2)}} \, \dot{D}$$

$$\dot{J} = \dot{M}_1 M_2 \sqrt{\frac{GD}{(M_1+M_2)}} + M_1 \dot{M}_2 \sqrt{\frac{GD}{(M_1+M_2)}} + \frac{1}{2} M_1 M_2 \sqrt{\frac{G}{D(M_1+M_2)}} \dot{D}$$

Divide through by J:

$$\frac{\dot{J}}{J} = \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} + \frac{1}{2}\frac{\dot{D}}{D}$$

We note that $\dot{M}_1 = -\dot{M}_2$:

$$\frac{\dot{J}}{J} = -\frac{\dot{M}_2}{M_1} + \frac{\dot{M}_2}{M_2} + \frac{1}{2}\frac{\dot{D}}{D}$$

$$= \dot{M}_2\left(\frac{1}{M_2} - \frac{1}{M_1}\right) + \frac{1}{2}\frac{\dot{D}}{D}$$

$$= \frac{\dot{M}_2}{M_2}\left(1 - \frac{M_2}{M_1}\right) + \frac{1}{2}\frac{\dot{D}}{D}$$

$$\therefore \frac{\dot{D}}{D} = 2\frac{\dot{J}}{J} + \frac{2(-\dot{M}_2)}{M_2}\left(1 - \frac{M_2}{M_1}\right)$$

For conservative mass transfer from M_2 to M_1 we have that $\dot{M}_2 < 0$ and ~~[illegible]~~ $\dot{J} = 0$

$$\boxed{\frac{\dot{D}}{D} = \frac{2(-\dot{M}_2)}{M_2}\left(1 - \frac{M_2}{M_1}\right)}$$

$$\frac{\dot{D}}{D} = \frac{2|\dot{M}_2|}{M_2}\left(1 - \frac{M_2}{M_1}\right) \quad \text{for } \dot{M}_2 < 0$$

Thus, we see that the stellar separation increases as a result of mass transfer if $M_2 < M_1$, so mass transfer from a less massive object to a more massive one results in a widening of the orbit. Eventually mass transfer would stop when the Roche lobe radius exceeds the size of the donor star, unless stellar evolution can cause the donor to expand. Mass transfer from a more massive donor star, on the other hand, causes the orbit to shrink, thus maintaining the accretion process by allowing Roche lobe overflow to persist. But this situation may change if mass transfer eventually causes $M_2 < M_1$.