Extrasolar Planets and Astrophysical Discs - Course work 6 Solutions

1. Centre of Mass $\underline{r}_{cm} = \dfrac{m_1 \underline{r}_1 + m_2 \underline{r}_2}{m_1 + m_2}$.

Assume that $\underline{r}_{cm} = 0$

$\Rightarrow m_1 |\underline{r}_1| = m_2 |\underline{r}_2|$

$\therefore |\underline{r}_1| = \dfrac{m_2}{m_1} |\underline{r}_2|$.

The Velocity of Mass m_1 as it orbits around the Centre of Mass is

$$v_1 = r_1 \Omega \qquad \text{where } \Omega = \sqrt{\frac{G(m_1 + m_2)}{D^3}}$$

= Orbital angular Velocity.

$D = |\underline{r}_2 - \underline{r}_1|$.

= orbital Separation.

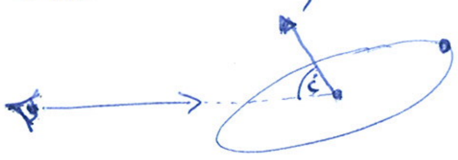
$$\therefore v_1 = \frac{m_2}{m_1} r_2 \Omega .$$

For $m_1 >> m_2$, as is the case if m_2 is a planet, $\Omega \simeq \sqrt{\dfrac{G m_1}{D^3}}$ where $D \simeq r_2$ when $m_1 >> m_2$

$$\therefore \quad v_1 = \frac{m_2}{m_1} D \sqrt{\frac{GM_1}{D^3}} = \frac{m_2}{m_1} \sqrt{\frac{GM_1}{D}}.$$

$$\therefore \quad m_2 = m_1 v_1 \left(\frac{GM_1}{D}\right)^{-1/2}$$

v_1 is the observed value of the stellar radial velocity due to its motion around the centre of mass. As we do not know the orientation of the planet-star orbital plane relative to the line-of-sight, the observed value of $v_1 = v_{real} \times \sin i$, where i = inclination angle



$$\therefore \quad \boxed{m_2 \sin i = m_1 v_{rel} \sin i \left(\frac{GM_1}{D}\right)^{-1/2}}$$

We obtain D from the orbital period and the stellar mass:

$$P = \frac{2\pi}{\Omega} = 2\pi \sqrt{\frac{D^3}{GM_1}}$$

$$\therefore \quad \boxed{D = \left(\frac{P}{2\pi}\right)^{2/3} (GM_1)^{1/3}}$$

For Solar-type Star $M_1 = 2 \times 10^{30}$ kg.

$P = 300$ days $= 2.592 \times 10^7$ Secs.

$G = 6.67 \times 10^{-11}$.

$\therefore D = 1.31 \times 10^{11}$ m.

$$M_2 \sin i = 2 \times 10^{30} \times 1 \times \left(\frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{1.31 \times 10^{11}} \right)^{-1/2}$$

$$= 6.27 \times 10^{25} \text{ kg}$$

$$\simeq 10.5 \text{ Earth Masses.}$$

Dimming of luminosity $\frac{\Delta L}{L_*} = 10^{-4}$.

$\therefore$ Since the luminosity dimming depends on the ratio of $\frac{\pi R_p^2}{\pi R_*^2}$, $R_p^2 = 10^{-4} R_*^2$

$\Rightarrow R_p = 10^{-2} R_* = 10^{-2} \times 7 \times 10^8$ m.

$\therefore$ $\underline{R_p = 7 \times 10^6 \text{ m.}}$

Now we calculate the planet density

$$\rho_P = M_P \frac{3}{4\pi R_P^3} = 4.4 \times 10^4 \text{ kg m}^{-3}$$

$\simeq$ 40 times more dense than water.

Such a planet would have to be composed of a very dense material - actually twice as dense as Uranium!

$$\text{Energy absorbed} = \frac{L_*}{4\pi a^2} \times \pi R_P^2$$

a = semi-major axis.

$$\text{Energy emitted} = 4\pi R_P^2 \sigma T_{eff}^4 .$$

$$\therefore T_{eff}^4 = \frac{L_*}{16\pi a^2 \sigma}$$

From Keplers 3rd Law: $P^2 \propto a^3$

$\Rightarrow a = 1.5 \times 10^{11} \left(\frac{300}{365}\right)^{2/3}$
$= 1.31 \times 10^{11}$.

$\sigma = 6 \times 10^{-8} \text{ J s}^{-1} \text{m}^{-2} \text{K}^{-4}$.

$L_* = 4 \times 10^{26} \text{ J s}^{-1}$

$$\therefore T_{eff} = \left[\frac{4 \times 10^{26}}{16 \times \pi \times (1.31 \times 10^{11})^2 \, 6 \times 10^{-8}}\right]^{1/4}$$

$= 295.8 \text{ K.} \simeq 22°\text{C}$ - In habitable zone!!

Temperature differences: Albedo due to clouds or reflective surface features; atmosphere with greenhouse gases.