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CONJUGACY CLASS REPRESENTATIVES IN THE MONSTER GROUP

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Abstract

We describe a procedure for determining (up to algebraic conjugacy) which conjugacy class any element of the Monster lies in, using computer constructions of representations of the Monster in characteristics 2 and 7. We use this procedure to calculate explicit representatives for each conjugacy class.

1. *Introduction*

The Monster group, $\mathbb{M}$, is the largest of the 26 sporadic simple groups, having order

$$2^{46}.3^{20}.5^{9}.7^{6}.11^{2}.13^{3}.27.19.23.29.31.41.47.59.71$$
$$= 808,017,424,794,512,875,886,459,904,961,710,757,005,754,368,000,000,000$$

It is useful to find conjugacy class representatives in groups, and this has been done explicitly for the other 25 sporadic simple groups (see [**4, 7**]), but the Monster is more difficult to deal with. The main obstacle to effective computation in the Monster is the large degree of the smallest faithful representation: 196883 over $\mathbb{C}$ or a mere 196882 over $\mathbb{F}_2$ or $\mathbb{F}_3$. We make use of the original computer construction of the Monster [**3**] and of an analogous construction over $\mathbb{F}_7$ due to the second author [**6**] to find words in the generators for an element in each conjugacy class of $\mathbb{M}$, up to algebraic conjugacy. We are not able to distinguish the classes of elements of order 27 as they have the same character values and power maps. Incidentally, these are precisely the same sets of classes whose McKay–Thompson series are identical [**1**]. However, we are able to produce an element in class 27*A* and another element in class 27*B*, and correspondign words, by a different method.

2. *Computer Constructions Of The Monster*

We recapitulate some useful information from [**3**]. The Monster is constructed from subgroups of shape $N(3B) = 3^{1+12}{\cdot}2{\cdot}\mathrm{Suz}{:}2$ and $3^{2+5+10}{\cdot}(\mathrm{M}_{11} \times 2{\cdot}\mathrm{S}_4)$ intersecting in a group of shape $3^{2+5+10}{\cdot}(\mathrm{M}_{11} \times \mathrm{S}_3)$. Importantly for us, the 196882 dimensional module over $\mathbb{F}_2$ decomposes under the action of $N(3B)$ into submodules either of small dimension or with useful structure. This means we can compute with relative ease in this group. Our generators for $N(3B)$ are B, C and E where $\langle B,C\rangle = 6{\cdot}\mathrm{Suz}{:}2$ and are standard generators in the sense of [**8, 5**], and E is a specified element of 3^{1+12}. A fourth element T, of order 2, extends a suitable subgroup $3^{2+5+10}{\cdot}(\mathrm{M}_{11} \times \mathrm{S}_3)$ to $3^{2+5+10}{\cdot}(\mathrm{M}_{11} \times 2{\cdot}\mathrm{S}_4)$. We have $\langle B,C,T\rangle = \langle B,C,E,T\rangle = \mathbb{M}$, but it is not possible to multiply T with any of the other 3 generators. To work in the Monster we are reduced to acting on vectors by words of the

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form $W_1 T W_2 T \ldots$ for $W_i \in N(3B)$. Acting on a random vector allows element orders to be calculated with high probability, and using two vectors whose stabilisers intersect trivially increases this probability to 1. The trace of an element can be found by acting on each basis vector in turn and adding the appropriate entry of the result to a running total. The number of occurrences of Ts in a word is the main factor determining the speed of the calculation. On a Pentium III 700MHz computing the trace for a word $W_1 T$ takes about 30 minutes.

The construction in [**6**] uses the same generators B, C, E, and T to make the 196883-dimensional representation over $\mathbb{F}_7$. In this case, calculating the trace for a word $W_1 T$ takes about 7 hours.

3. *Implementation*

One reason why the modulo 2 construction is considerably faster is the efficiency of binary arithmetic on a binary computer. The programs that implement the $\mathbb{F}_2$ construction of [**3**] are based on R. Parker's C MeatAxe which stores an element of $\mathbb{F}_2$ as a single bit so that addition and multiplication can be done 32 elements at once (on a 32-bit computer) using the operations `xor` and `and` (respectively). With this in mind we modified the modulo 7 programs as follows. Originally each byte stored two field elements, say (a,b), as $7a+b$ and arithmetic was done with a 256×256 lookup table, two field elements at a time. In our modified packing we store a in the first nibble of the byte and b in the second *i.e.,* $32a+b$. Since $7+7<16$ we can safely add such bytes together as one nibble can never overflow into the next, *e.g.,*

$$\begin{aligned}(2,6)+(3,1) &\to \texttt{00100110}+\texttt{00110001} \\ &= \texttt{01010111} \\ &\to (5,7)\end{aligned} \qquad \begin{aligned}(2,6)+(6,6) &\to \texttt{00100110}+\texttt{01100110} \\ &= \texttt{10001100} \\ &\to (8,12)\end{aligned}$$

To perform the modular reduction on a byte we first add the byte `00010001` to make 7 overflow to 8 in each nibble. Now take the bitwise `and` of this byte and `10001000`, subtract it, shift it 4 places to the right, then add it on (this performs reduction modulo 8). Finally subtract `00010001`. *e.g.,*

$$\begin{aligned}\texttt{01010111}+\texttt{00010001} &= \texttt{01101000} \\ \texttt{01101000 and 10001000} &= \texttt{00001000} \\ \texttt{01101000}-\texttt{00001000} &= \texttt{01100000} \\ \texttt{00001000} \xrightarrow{4} &= \texttt{00000001} \\ \texttt{01100000}+\texttt{00000001} &= \texttt{01100001} \\ \texttt{01100001}-\texttt{00010001} &= \texttt{01010000} \\ &\to (5,0)\end{aligned} \qquad \begin{aligned}\texttt{10001100}+\texttt{00010001} &= \texttt{10011101} \\ \texttt{10011101 and 10001000} &= \texttt{10001000} \\ \texttt{10011101}-\texttt{10001000} &= \texttt{00010101} \\ \texttt{10001000} \xrightarrow{4} &= \texttt{00010001} \\ \texttt{00010101}+\texttt{00010001} &= \texttt{00100110} \\ \texttt{00100110}-\texttt{00010001} &= \texttt{00010101} \\ &\to (1,5)\end{aligned}$$

This procedure is considerably slower than the old lookup table. However, we can apply it to 32 bits at once instead of just bytes, thus processing 4 times as many field elements at once and realising approximately a two-fold speed increase.

4. *Conjugacy Class Representatives*

4.1. *Distinguishing Conjugacy Classes*

The ordinary character of degree 196883 is rational and therefore cannot be used to distinguish algebraically conjugate classes. Moreover, the classes $27A$ and $27B$ have the same character value and power maps, so these classes also cannot be distinguished in this way. It is straightforward to check that, up to these ambiguities, the conjugacy class of an element is determined by its order and the character values of it and its powers. Indeed, it is sufficient to use the reduction of the character value modulo 14, *i.e.*, mod 2 and mod 7.

We obtain our character values from trace calculations performed on explicit elements of $\mathbb{M}$. Note that modulo 2 the 196883-dimensional module splits as $1+196882$ and so the traces obtained from calculations have the other parity to the character values tabulated in the ATLAS for χ_2. Modulo 7 there is no such splitting and the traces are the 7-modular reductions of the values of χ_2. Wherever possible the modulo 2 values are used since the calculations are much quicker.

For each order of element in $\mathbb{M}$ we tabulate the traces we must find in order to identify each class of elements of that order. These data allow us to answer a second question: Given an element of order n what information must we calculate in order to determine its conjugacy class?

These "conjugacy class identification data" are displayed in Table 1. The format is easily read by machine (thus is a little cryptic). For each order n of element in $\mathbb{M}$ there is a header row followed by one line for each class of elements of order n. The header row shows what needs to be calculated: "2" for a trace modulo 2, "7" for a trace modulo 7, and "`aPb`" for the trace modulo `a` of the `b`th power. The subsequent rows give the traces for the class indicated in the first column. A header row containing an asterisk indicates that classes of order n cannot be distinguished. A header row containing the number `1` indicates that there is only one conjugacy class of elements of order n, so no traces need to be calculated.

4.2. *Searching For Conjugacy Class Representatives*

In exact analogy with finding conjugacy class representatives in other sporadic simple groups [**4**], a random search is conducted. The strategy is to generate a long list of words, calculate the orders of the corresponding elements of $\mathbb{M}$, and then use the conjugacy class identification information to work out in which classes these elements lie.

It is sufficient to find a generator for each conjugacy class of maximal cyclic subgroup *i.e.*, a representative for every class that cannot be obtained by powering up an element of greater order. Excluding the classes of algebraically conjugate elements these are $18E$, $24A$, $24C$, $24D$, $24E$, $24F$, $24G$, $24H$, $24I$, $24J$, $30A$, $30F$, $32A$, $32B$, $36A$, $36B$, $36C$, $36D$, $38A$, $39B$, $40A$, $40B$, $41A$, $42D$, $45A$, $48A$, $50A$, $51A$, $52B$, $54A$, $56A$, $57A$, $60A$, $60B$, $60C$, $60D$, $60E$, $60F$, $66A$, $66B$, $68A$, $70A$, $70B$, $78A$, $78C$, $84A$, $84B$, $84C$, $105A$, $110A$. So we must definitely find representatives for these classes. However, powering up produces words with many Ts which are difficult and slow to work with. Therefore it it preferable to find representatives for as many classes as possible, not just those whose elements generate maximal cyclic subgroups. Of course whenever possible it is preferable to find words in $\langle B,C,E\rangle$ since they are much faster to calculate with.

4.3. *Word Generation Strategies*

Product replacement [**2**] has been shown to produce genuinely random words in exactly our situation, but the word length soon becomes very large which would make our calcu-

```
2   7
B   2
A   3

3   7
B   4
A   5
C   6

4   7   7P2
D   1
A   2   2
B   2   3
C   5

5   2
A   0
B   1

6   7   2
B   0   0
C   0   1
A   1
D   4
E   5
F   6

7   2
B   0
A   1

8   7   7P2
A   0
D   2
E   3
B   4
F   6   1
C   6   2

9   2
A   1
B   0

10  7   2
A   0   0
E   0   1
C   3
B   5
D   6

11  1

12  7   2   7P2 7P3
A   0
I   1
G   4
F   5   0   0
B   5   0   5
E   5   1
J   6   0       1
D   6   0       2
H   6   0       5
C   6   1

13  2
A   0
B   1
```

```
14  2   7
C   0
B   1   2
A   1   3

15  2   2P3
A   0   0
D   0   1
B   1   0
C   1   1

16  7
A   3
B   6
C   1

17  1

18  2   2P3 7
A   0       4
E   0       5
D   0       6
B   1   0
C   1   1

19  1

20  7   2
A   5
B   1
C   0
D   4   0
E   2
F   4   1

21  2   2P7 7
C   0   0
A   0   1
D   1       4
B   1       5

22  7
B   0
A   4

23  *

24  7   7P2 7P3
H   1
A   2   0
B   2   5
I   3
D   5
C   6   1
J   6   6   6
G   6   4
F   6   5
E   6   6   4

25  1

26  2
A   0
B   1

27  *
```

```
28  2   7   7P2
D   0
B   1   2   2
A   1   2   3
C   1   5

29  1

30  2   2P3 7   7P3
B   0   0   2
C   0   0   6
E   0   1
D   1   0
G   1   1       0
F   1   1       3
A   1   1       6

31  *

32  7P2
B   1
A   6

33  2
B   0
A   1

34  1

35  2
B   0
A   1

36  7
C   0
D   1
A   2
B   6

38  1

39  2   2P3
A   1
B   0   0
CD  0   1

40  2   7
A       6
B       1
CD 1

41  1

42  2   2P7
C   0   0
A   0   1
B   1   0
D   1   1

44  *

45  1

46  7
CD  1
AB  6

47  *
```

```
48  1

50  1

51  1

52  2
A   0
B   1

54  1

55  1

56  2
BC  0
A   1

57  1

59  *

60  2   2P3 7
A   0   0   1
B   0   0   6
F   0   1
E   1   0
C   1   1   0
D   1   1   5

62  *

66  2
A   0
B   1

68  1

69  *

70  2
B   0
A   1

71  *

78  2
BC  0
A   1

84  2   2P7
C   0   0
A   0   1
B   1

87  *
88  *
92  *
93  *
94  *
95  *
104 *
105 1
110 1
119 *
```

Table 1: Conjugacy class identification data file. The data shows which traces to calculate in order to determine the conjugacy class of an element.

lations too slow. Instead we conducted a search through words of the form W_1T and W_2 where W_1 is a word in B and C, and W_2 is a word in B, C, and E. Eliminating words containing B^3, C^4, or E^3 we obtained $1 \cdot 4$ million words of shape W_1T and calculated element orders for 160,000 of these. For each element order we produced trace calculation jobs according to the data in Table 1. We store the results of all these calculations in a database which is used to produce further jobs until we have witnessed all classes of elements of that order. For the elements of order 12 for example we ran the following jobs:

1. Trace modulo 7.
2. If the trace modulo 7 is 5 or 6 then calculate the trace modulo 2.
3. If the trace modulo 7 is 5 and the trace modulo 2 is 0 then calculate the trace modulo 7 of the square.
4. If the trace modulo 7 is 6 and the trace modulo 2 is 0 then calculate the trace modulo 7 of the cube.

Representatives for most of the conjugacy classes were found in this way using words of the form W_1T. A few classes of elements of orders 24 and 36 did not turn up and required more sophisticated methods. These classes were $24C$, $24G$, $24F$, $24J$, and $36B$. In fact only 43 elements of the form W_1T have been found in classes $24J$, $24G$, $24F$ and $24E$ (most of them in $24E$) but finding these took over 60 days of computer time.

4.4. *Example: Finding An Element In Class* $24C$

Let $x \in 24C$ then $x^8 \in 3B$ so $C_{\mathbb{M}}(x)$ is the centraliser in $3^{1+12}{\cdot}2{\cdot}\mathrm{Suz}$ of x^3. Since x^3 has order 8 we may think of it as being an element of $2{\cdot}\mathrm{Suz}$, which has the following conjugacy classes of elements of order 8:

- Class $8A$ with centraliser in $2{\cdot}\mathrm{Suz}$ of order $192 = 2^6.3$.
- Classes $+8B$ and $-8B$ both with centraliser in $2{\cdot}\mathrm{Suz}$ of order $2.64 = 2^7$.
- Class $8C$ with centraliser in $2{\cdot}\mathrm{Suz}$ of order $32 = 2^5$.

Since $|C_{\mathbb{M}}(x)| = 3456 = 2^7.3$ and 3^{1+12} is a 3-group, the 2-powers in these centraliser orders show that x^3 must lie above class $8B$ of Suz.

Now, C and $B = D$ are standard generators for $6{\cdot}\mathrm{Suz}{:}2$ and standard generators for $6{\cdot}\mathrm{Suz}$ are A and B where

$$A = (CDD)^{-2}(CD)^{14}(CDD)^2$$

as in [**3**]. Using A and B in the recipe in [**8**] for the conjugacy classes of Suz yields elements of $6{\cdot}\mathrm{Suz}$ lying above the corresponding classes of Suz. From this we obtain elements g_8 lying above $8B$ of order 8, and g_{10} lying above $10B$ of order 60. Therefore $g_6 = g_{10}^{10}$ is a central element of order 6 in $6{\cdot}\mathrm{Suz}$. To find all classes lying above $8B$ we calculate the order and traces for $g_6^i g_8$ for $0 \leqslant i \leqslant 5$ and thus find elements of $\mathbb{M}$ in classes $8A$, $8C$, $24C$, and $24I$.

As a word in B and C the length of the element in $24C$ is 161455 which is too long for our results table and takes about 30 minutes to make on machine (using a naïve algorithm). In A and B this is reduced to 205. Looking at elements lying above class $8C$ of Suz yields words in $24G$ and $8C$, but we already have good words for these classes.

4.5. *Elements of Order 27*

Class $27A$ has the same character values and power maps as does class $27B$, so they cannot be distinguished mechanically using our method. However, $|C_{\mathbb{M}}(27A)|$ is divisible by 2 whereas $|C_{\mathbb{M}}(27B)|$ is not, and we can exploit this to find words in $\langle B,C,E\rangle$ for elements in each of these classes.

To find such elements we work in the three-fold cover G of $N(3B)$. This is a group of shape $G \cong 3^{1+12}{:}6^{\cdot}\mathrm{Suz}{:}2$ and it has a representation in 38 dimensions over $\mathbb{F}_3$ that was constructed in [**3**]. Using this representation we find words in $\langle B,C\rangle$ for an element g of order 9 in $6^{\cdot}\mathrm{Suz}{:}2$. There are 3 such conjugacy classes in $6^{\cdot}\mathrm{Suz}{:}2$ and they all project onto the same conjugacy class in $2^{\cdot}\mathrm{Suz}{:}2$, so we may choose whichever we like.

Now extending downwards to G, we find two conjugacy classes of elements of order 27, and four classes of elements of order 9. One of these classes of elements of order 27 must become $27A$ in the Monster and the other must become $27B$. This is because these two classes are the only two classes of elements of order 27 in the quotient group $N(3B)$. We compute $C_G(g)$ to find which is centralised by an involution, and thus distinguish the classes.

Straight line programs for these elements are included in the archive.

Note that an element in class $27A$ can also be obtained by squaring an element in class $54A$.

5. *Results*

The results consist of a list of conjugacy class names with corresponding words. The table below gives for each conjugacy class of $\mathbb{M}$ the shortest word we found for an element in that class.

The class names are in the second column, and in the third column is the number of words found in the class, if there is a sum then the summands give the number of words of the form W_1T and W_2 respectively. The fourth column gives either a word, or where no element has been found the name of a class which powers up to the particular class, followed in brackets by the number of Ts the power has. Any element that powers up to class $3B$ is conjugate to an element in $\langle B,C,E\rangle$, as are all other powers. Such conjugacy classes are indicated by a bullet in the first column. Of course this test does not identify all of the conjugacy classes of $\mathbb{M}$ that are represented in $\langle B,C,E\rangle$.

Table 2: Words for conjugacy class representatives

$\langle B,C,E\rangle$	Class	Number	Word
•	$2B$	8	$6C(0)$
•	$2A$	0	$4B(0)$
•	$3B$	0	$6D(0)$
	$3A$	0	B
	$3C$	0	$BCCCBBCBCBCBCBCBBCBBC-$ $-BBCCCBBCBCBCBCBCBBCBBC-$ $-BCBBCBBCBBCBBC$
•	$4D$	6982	$BCCCBB$
•	$4A$	8	$BCCBBT$ or $24A(0)$
•	$4B$	84	$BCCCBBCBCBCBCBCBBCBBCB$
•	$4C$	0	$8D(0)$
•	$5A$	14	$BCCCBCCBCBBCBBCBCBCBCBC$
•	$5B$	0	$10C(0)$
•	$6B$	0	$12F$, $18A$, $18D(3)$, $18E(3)$

continued...

...continued

$\langle B,C,E\rangle$	Class	Number	Word
	$6C$	$1052+27$	$BCCB$
	$6A$	0	$12C(2)$, $18B$, $18C$
•	$6D$	0	$12G(0)$
•	$6E$	2270	$ECCB$
	$6F$	0	$12D(4)$
•	$7B$	118	$BCCCBCCBC$
	$7A$	0	$14A$, $14B$, $21A(3)$, $21C$
•	$8A$	0	$16A$, $24B$ and see page 5 of text
•	$8D$	0	$24D(0)$
•	$8E$	0	$16B$, $16C$, $24I(0)$
	$8B$	14	$BBCBBCCBCBCBBCBBCBCCCBCBCT$ or $24A(0)$
•	$8F$	522	$BCCCBCBCBCBCBCBC$
•	$8C$	573	$BCCCBCBCBBCBCBBCBCBC$
•	$9A$	0	$18B(2)$
•	$9B$	55	$BCCCBCCBCBCBCBCBCB$
	$10A$	0	$20B$, $30B(3)$
•	$10E$	0	$20C$, $20F(2)$, $30G$
•	$10C$	170	$BCCCBCBCBCBBCBCBCBCB$
•	$10B$	6383	$BCCCBCBBCBCBCBBCBC$
•	$10D$	0	$20E(2)$
•	$11A$	0	$22A$, $22B(0)$
	$12A$	0	$24A(0)$, $60B$
•	$12I$	368	$BCCCBCBCBCCCBCBBCBBCBCB$
•	$12G$	105	$BCCCBCBBCBCBCBBCBCBBCB$
•	$12F$	0	$24F(2)$, $60E$
•	$12B$	7	$BBCBBCBBCBCBCBBCCCBCBCBT$
	$12E$	0	$24D(0)$
	$12J$	0	$24J(2)$, $60F$
	$12D$	0	$24E(2)$
•	$12H$	$0+2$	$24H(0)$
	$12C$	419	$BCCBCCBCBCCCT$
	$13A$	0	$26A(2)$
•	$13B$	54	$BCCCBCCCBCBCBCCBCBCBCBC$
•	$14C$	$116+2$	$BCCCBCCCBCBC$
	$14B$	0	$28B(2)$
	$14A$	0	$28A$, $42A(3)$
	$15A$	0	$30B(2)$
	$15D$	0	$30E(2)$
•	$15B$	0	$30D(2)$
•	$15C$	3	$BCBCBCBCBBCBBCBCT$
	$16A$	$48+1$	$BCCCBBCBCBCBCBCBBCBBC-$ $-BBCCCBBCBCBCBCBCBBCBBC-$ $-BBCCCBCBCBBCBCBCBCBCBBCB$
	$16B$	0	$32A(2)$
	$16C$	0	$32B(2)$
	$17A$	55	$BCBCBCBCBCBCBCBT$
•	$18A$	0	$54A(3)$
•	$18E$	26	$BBCBCBCBBCBCCT$

continued...

...continued

$\langle B,C,E\rangle$	Class	Number	Word
•	18*D*	48	*BCBCBCBCCCBBT*
•	18*B*	57	*BCBBCBBCBCBBT*
•	18*C*	0	36*D*(2)
	19*A*	21	*BBCBBCBBCBBCCCBBCBCBCBCBBT*
	20*A*	0	40*B*(2)
	20*B*	0	40*A*(2)
•	20*C*	0	60*C*(3)
•	20*D*	12	*BCBCBBCBCBBCBCCT*
	20*E*	4	*BBCBBCBCBCBCBBCBCBBT*
•	20*F*	4	*BBCBBCBCBBCBBCT*
	21*C*	0	42*C*(2)
	21*A*	3	*BBCBBCCBCBCBBCBCBCBBCBCT*
•	21*D*	1	*BBCBBCBBCBCBBCBCBCBCT*
	21*B*	1	*BBCBBCBBCBBCBBCBBCBCBBT*
•	22*B*	9	*BCCCBCBCBCBCBCBCBCBCBCBCB*
	22*A*	21	*BBCBBCBBCBCBT*
	23*AB*	2361	*BCCBCCT*
•	24*H*	623	*BCCCBCBCBCBCBBCBCBCBC*
	24*A*	1	*BCBCBBCBCBCBBCBCT*
			and see page 5 of text
	24*B*	1	*BCBCBBCBCBCBBCBCBT*
•	24*I*	31	*BBCBBCBCBCBCBCBBT*
	24*D*	$222+23$	*BCCCBCBCBBCBCBCBCBCBBCB*
•	24*C*	0	see page 5 of text
	24*J*	1	*BCBCBCBBCBCBCCCT*
•	24*G*	9923	*EBBCBCBCBCBC*
•	24*F*	9	*BBCBBCBCBCBBCBBCBCBT*
	24*E*	33	*BBCBCBCBCCCBT*
	25*A*	949	*BBCBBCBCBBCBBT*
	26*A*	8	*BBCBBCBCBCBCCCBCBBCBCCT*
•	26*B*	78	*BBCBCCCBCBCBBT*
•	27*A*	0	54*A*(2)
•	27*AB*	454	*BBCCBCBCBCBBCT*
•	28*D*	47	*BBCCCBCBCBCCBCBT*
	28*B*	27	*BBCCCBT*
	28*A*	0	84*A*(3)
	28*C*	31	*BCBBT*
	29*A*	1553	*BCCBCBCBBCBCT*
	30*B*	1	*BBCBBCCBCBCBBCBCBCBBCBCCCT*
	30*C*	5	*BBCBBCBCBCBCBCBCBBCBCBBBT*
	30*E*	107	*BCBCBBCBCBBCBBCT*
•	30*D*	2	*BBCBBCBCBCBCBT*
•	30*G*	1	*BBCBCBCBBCBBCT*
•	30*F*	13	*BBCCCBCBCT*
•	30*A*	1	*BCBBCBCBCBBCCT*
	31*AB*	3034	*BBCT*
	32*B*	3	*BCBBCCCT*
	32*A*	1	*BCBCBCBCCT*
	33*B*	5	*BBCBBCCBBCBBCBCBCBBCBCT*

continued...

...continued

$\langle B,C,E\rangle$	Class	Number	Word
•	33*A*	35	*BBCBCBBCBBCBCBT*
	34*A*	3221	*BCCBCBCBT*
	35*B*	1316	*BCBCBCCBCBT*
	35*A*	0	70*A*(2)
•	36*C*	3	*BCBCBBCBCBBCBBT*
•	36*D*	590	*BCBCBBT*
•	36*A*	1	*BBCCBCBCBCBCBCCCT*
•	36*B*	1	*BBCBBCBBCBBCBBCBCBCBBT*
	38*A*	1628	*BBCBCBCBCBT*
	39*A*	102	*BCBBCBCBCCT*
	39*B*	3	*BBCCBCBBCBBCBCT*
•	39*CD*	44	*BBCBCBBBT*
	40*A*	18	*BBCBBCBBCBCT*
	40*B*	1	*BBCBBCBCBBCBBCBCT*
	40*CD*	688	*BCCCBCBBT*
	41*A*	4949	*BCCT*
	42*C*	75	*BCBBCBCBBCBCBT*
	42*A*	1	*BBCBBCBBCCCBCBBCBCBCBCBT*
•	42*B*	1	*BCBCBCBBCBCBBCBCT*
	42*D*	43	*BCBCBCCBCBCT*
	44*AB*	1657	*BCBCBCBCBCBT*
•	45*A*	2049	*BCBT*
	46*CD*	21	*BBCCCT*
	46*AB*	5	*BBCBBCBCT*
	47*AB*	4670	*BCBCBCBBT*
	48*A*	617	*BCBBCBCBBCBBCT*
	50*A*	3342	*BBCBBCCCT*
	51*A*	1102	*BCBBCBCBBCT*
	52*A*	5	*BBCCCBCBCBCBCT*
	52*B*	33	*BBCBCBBT*
•	54*A*	2614	*BCBBCBCBBCBBT*
	55*A*	2035	*BBCBBCBCBBCBCT*
	56*BC*	8	*BBCBBCBCCBCBBCBBT*
	56*A*	6	*BBCCBBCBCBBBT*
	57*A*	1744	*BCCCT*
	59*AB*	3573	*BCBCBCBBCCCT*
	60*A*	4	*BCBBCBCBCCBCBCT*
	60*B*	1	*BCBCBCBCBCBBT*
	60*F*	80	*BBBCT*
•	60*E*	35	*BCBCCBCBCT*
•	60*C*	5	*BBCBCCCBCBBCT*
•	60*D*	2	*BCBCBCBCBCCCT*
	62*AB*	7404	*BCBCBT*
	66*A*	5	*BBCBBCBCBBCBCCBCBBCBCBCT*
•	66*B*	290	*BBCBCBBCBCBT*
	68*A*	2273	*BCBCBCBCT*
	69*AB*	3406	*BBCBCCCT*
	70*B*	3	*BBCBBCBCBBCBCBBCT*

continued...

...continued

$\langle B,C,E \rangle$	Class	Number	Word
	70A	41	$BBCBCBCBCT$
	71AB	2588	$BCBBCBCBCT$
•	78BC	14	$BBCBCBBCCBBBT$
	78A	30	$BBCBCBCBBBT$
	84C	2	$BBCBBCBBCBBCBCCCT$
	84A	3	$BBCBBCBBCBCBBCBCCT$
•	84B	8	$BBCBBCBBCBCCBBT$
	87AB	3521	$BCBBCBCT$
	88AB	2130	$BCBCBCCBCT$
	92AB	5731	$BBCBCBCBT$
	93AB	6910	$BCBBCBBBT$
	94AB	1452	$BCCCBCBCT$
	95AB	2313	$BCBCBBCBBBT$
	104AB	1367	$BCCCBCBCBCT$
	105A	888	$BBCCBCBCBCBCBCBT$
	110A	238	$BCBCBBCBCBCBCBBBT$
	119AB	3248	$BBCBBCBBBT$

6. *Monster Element Database*

Due to the large number of words being used it became obvious early on that some sort of database was needed. We store elements and their trace information in text files, one file for each order of elements in $\mathbb{M}$. A suite of Perl scripts manage the database, in particular they can determine what needs to be calculated and produce input data files for the trace finding programs.

The main database programs and data are included with this paper, see the `README.txt` file for an explanation of how to use it.

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