Three is equal to one

I promised to tell you the story of how I came to believe that three is equal to one, and what extraordinary consequences this led to. It all started when I was sent, for my education, at an impressionable age, to an institution whose very name proclaimed to the world that three is equal to one. At first I did not take this too seriously. I *knew* that three was not equal to one. It was an article of almost religious faith for me, and I never questioned it. Worse, I never even admitted that it *could* be questioned.

But there is something insidious about hearing intoned everyday the mantra "Three is one and one is three", even if you don't believe it, and think you will never believe it. The thought worms its way into your head and eats away at the unshakable certainties in your mind. The certainties turn out not to be so unshakable after all. And before you know it, you too are intoning "Three is one and one is three", and perhaps you start to believe it a little also.

So let's take this assertion, as we must, as an article of religious faith, and see what else the followers of this faith must believe. For if $1 = 3$ then by adding 1 to both sides we get $2 = 4$. Adding 1 again gives $3 = 5$. And again to get $4 = 6$, $5 = 7$, and so on. So we have $1 = 3 = 5 = 7 = 9 = \dots$ and $2 = 4 = 6 = 8 = 10 = \dots$. What about *subtracting* 1 from both sides of the equation $1 = 3$? We get $0 = 2$, and then $-1 = 1$, and $-2 = 0$, and $-3 = -1$, and so on and on. Therefore all the odd numbers are equal to 1 and all the even numbers are equal to 0.

These religious zealots (let us call them dualists) have laid waste to our beautiful number system, but you have to admit that they have it easy when it comes to learning their times tables: "one one is one" and that's it! Why, you could skip half a year of primary school by not having to learn your times tables. Perhaps there is something to be said for this religion after all.

If $2 = 0$, you might have thought that whenever two teachers were in the same room at the same time they would instantly cancel each other out and disappear in a puff of smoke. Unfortunately for us, this did not seem to happen. Strangely enough, though, this obvious contradiction with their religious beliefs did not in any way diminish the dualists' conviction that $1 = 3$. All we got by way of explanation were remarks to the effect that "it is a mystery too deep for mere mortals to understand". That was on a good day. Usually it was "shut up and write out 1000000 times 'I must not be cheeky to the teachers'."

A million lines! Oh no! Actually, it wasn't that bad, because it wasn't really a million. I forgot to explain to you the dualists' code. Even they realised that however hard they maintained that $1 = 3$, this did pose problems when they wanted to count beyond two. So they devised a code, and instead of counting $1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots$ they counted like this: 1, 10, 11, 100, 101, 110, 111, 1000, 1001, ... And the rule they used for constructing this code was very simple: instead of the heavenly truth that $1 + 1 = 0$ they used a more down-to-earth version which said that $1+1$ is "0 carry 1", which meant that they wrote down 0 as usual but then carried a 1 into the next place to the left. Or, in ordinary language, $1 + 1 = 10$.

They found that they could do arithmetic in code as well. All they had to do was remember this carrying rule. Here's one for you:

$$\begin{array}{rcccc} & & & 1 & 1 \\ + & & 1 & 1 & 0 \\ \hline = & 1 & 0 & 0 & 1 \\ \hline & 1 & 1 & & \end{array}$$

(You would translate this as $3 + 6 = 9$, which you may think is easier—but did you *work out* that $3 + 6 = 9$, or did you just *remember* it?) They could do multiplication too:

$$\begin{array}{rcccc} & & 1 & 0 & 1 \\ \times & & & 1 & 1 \\ \hline = & & 1 & 0 & 1 \\ + & 1 & 0 & 1 & \\ \hline = & 1 & 1 & 1 & 1 \\ \hline \end{array}$$

or, as you would say, $5 \times 3 = 15$. And $1000 \times 1000 = 1000000$, so now you know how many lines we had to write, don't you?

Once we'd mastered this new way of doing arithmetic, we realised it was much easier than the normal way, but also much more boring. So we devised a way of relieving the tedium. We invented a machine which did it all for us. There were three of us in the original group: me, the thinker; Steve, the designer; and Alan, the practical guy. (You're probably going to ask me why there wasn't a girl in our group. Well, the girls thought what we doing was cheating, and they wanted nothing to do with it. Actually, that's not the real reason—the real reason is that we didn't know any girls.)

First we devised an adding machine. We realised that the essential component of the machine was a device to do the carrying. We decided to use a marble to represent a '1', and the absence of a marble to represent a '0', and we had a row of little wooden spoons to put the marbles in so they wouldn't roll away. So we had to devise a mechanism so that if two marbles fell into the same spoon, the spoon would tip up, and one marble would roll into the next spoon to the left, while the other one rolled away.

Fastening the spoons to a long rod so that they would pivot nicely was easy. We fastened the ends of the rod to the ends of a wooden box, and weighted the spoon handles so that they balanced the weight of one marble in the spoon. We put another rod underneath the spoon handles so they wouldn't overbalance when there was no marble in the spoon. Now we had a device for representing a number in a row of spoons, some of which had marbles in them. Of course, we want to add *two* numbers, so we had to have another row of spoons to represent the second number. We put the second row of spoons above the first one, so that each spoon could be tipped up, throwing its marble into the corresponding spoon in the lower row.

So far so good, but we weren't making much progress with the "carrying" mechanism. To tell the truth, we were having trouble getting the marbles to go where we wanted them to go. So we started to fill up the box with funnels and tubes to direct the marbles to the right places. But we still couldn't persuade two marbles in the same spoon to go in different directions.

So we tried a different approach. We let both marbles roll away, and got a new marble to do the carrying. We put a tube of new marbles above each spoon, closed at the bottom with a little lever. And when two marbles rolled onto one spoon, they would push the spoon handle up, which would release the lever above the next spoon to the left, just enough to let out one marble. Now we had our carrying mechanism, and we could build our prototype adding machine.

With mounting excitement we prepared to test the machine. We primed the bottom row of spoons with two marbles to represent the number 101 (five, to you and me), and the top row with three marbles to represent the number 111 (seven, to you and me). We turned the top row of spoons so the marbles fell into the bottom row. Two spoons now had two marbles each, and the carrying mechanism kicked into action, depositing two more marbles into the second and fourth spoons from the right. Now the second spoon had two marbles in it, so it too tipped up and released another marble into the third spoon. And when all the banging and clattering was finished, there was the answer, plain to behold: 1100, in the bottom row of spoons. There were also six spent marbles, rolling around in the bottom of the box.

Anyway, to cut a long story short, we made lots of improvements to our adding machine until we were satisfied that we had a machine which was efficient and reliable. Then we tried our hand at multiplication.

That's when the fun really started. We knew from our practice at doing multiplications by hand that it consisted of two parts—shifting and adding. Basically we had to take lots of copies of the first number, shifted by various distances (according to where the 1s were in the second number) and then add them all up. Well, we'd mastered adding up, so that was OK. We kept the bottom row of spoons for building up the answer gradually, and the row above was for the things we were adding in, one at a time. Remember these are just shifted copies of the first number we thought of. So we needed another row of spoons for the second number.

No sooner said than done: Alan made a third row of spoons for us. We didn't know where we were going to put it yet, but we knew we were going to need it. Then we hit a snag: every time we added in a number, that number disappeared. We couldn't add it in again, shifted or otherwise. So we decided we needed a mechanism for *copying* a row of marbles before it disappeared. Steve and I got together on this problem, and we spent ages trying to persuade these marbles to procreate. Of course, procreation is not a normal behaviour of marbles, so this was not as easy as it sounds. We even briefly considered using small animals or plants instead of marbles, but rapidly came to the conclusion that the potential benefits were outweighed by the disadvantages.

Anyway, we eventually came up with a design involving three rows of spoons arranged so that when the top row was released, the spoons which tipped up released *two* levers, thereby making copies of the original number in both of the new rows of spoons. Then Steve got together with Alan to make this new device, and they realised that the first row of spoons did not need to be above the other two. They even realised that if you were really careful you could get one of the levers to deposit the marbles back into the *first* row of spoons. All you had to do was make sure the spoons had returned to their original positions, with their catches on, by the time the new marbles fell into them. When we'd finally got this design working, we were feeling really pleased with ourselves.

We were really on a roll by this stage. By angling the levers so that they pointed to the next spoon along, we could get one (or both) of the copies to shift along by one spoon. Now we had all the ingredients we needed. Each step of the multiplication involved first shifting the second number to the right: if a marble dropped off the end, it controlled a lever which made the machine add the first number into the answer. Then we shifted the first number left, and repeated the whole process as many times as necessary. We had our multiplication machine!

Well, not quite. When it had finished, we found some marbles had fallen off the left hand end of the machine. We weren't really sure whether this mattered or not, so we did some experiments, and checked the answers given by the machine against the ones we got by doing the calculations ourselves. It took us a long while to figure out the pattern, but eventually we thought we had it: if all the marbles from the second number had fallen off the right-hand end *before* the marbles from the first number started falling off the left-hand end, then we were alright. Otherwise it seemed we were in trouble.

No-one really wanted to sit and watch the machine all the time to check whether the marbles fell off in the right order. (Well, not after the first few times, anyway—the novelty wore off rather quickly.) So we worked on a design for the machine to work this out for itself. The basic idea was to get it to stop working once the last marble from the second number had fallen off the right-hand end. Then we could look at the left-hand overflow to see if there were any marbles there—if so, then the calculations were too hard for our machine and we knew it had given us the wrong answer. But how could we get it to stop work? The whole thing was controlled by gravity, and we could hardly tell gravity to stop working!

There was another problem: how could the machine tell if the second row of spoons was empty? Actually, this wasn't so difficult. Remember this number is being shifted to the right one step at a time, and when we do the shifting we empty the whole row and then replace the marbles one place to the right. So we intercepted the new marbles on the way down: if there was any marble coming down, we kept it waiting while the adding was going on, and then let it go to trigger the next shift left of the other number. Otherwise, nothing triggered the next shift left, and the machine stopped.

Eureka!

From this point on, we never had to do any arithmetic again. We could just load up our machine with marbles, turn the handle, and out would pop the answer. After a while, the other students started to see the advantages, and wanted to use our machine themselves. We decided to rent it out by the hour, and charged a high price for it. We could afford to—there was no competition.

This was the beginning of our downfall, however. The teachers began to get suspicious when everyone was getting all their sums right. They launched an investigation, and found our machine. We were expelled from school and our machine was consigned to the bonfire.

Did I say downfall? Actually it was the best thing that ever happened to us. They could destroy our machine, but they couldn't destroy what was in our heads. So we built another one. And another. And another. Ans we sold all three. We called it the Miracle Multiplication Machine, or MMM for short, and we set up a company to make them and sell them. The orders came flooding in, and we had to employ dozens of people to make them for us. In a very short time we became rich beyond our wildest dreams.

So, you see, truth isn't everything, even in mathematics. Sometimes it pays to build on an obviously false hypothesis.