Peter Cameron: *The power graph of a group*

Let G be a group. The *directed power graph* of G is the graph with vertex set G, in which there is an edge from x to y whenever y is a power of x. The *undirected power graph* is the same graph but without directions, so x and y are joined if one is a power of the other.

This concept was developed by various Indian mathematicians including Shamik Ghosh, who asked at the British Combinatorial Conference this year whether the undirected power graph of a finite group determines the group up to isomorphism. This is false for groups in general, but true for finite abelian groups. We conjecture that if two finite groups have isomorphic power graphs, then they have the same numbers of elements of each possible order. (The conjecture is true for the directed power graph.) It is also true that the only finite group with the property that the group and its undirected power graph have the same automorphism groups is the Klein group of order 4.

I hope to explain all of this starting from the beginning, without assuming too much knowledge of group theory.