Queen Mary
University of London

MTH714U/MTHM024 Group theory

Assignment 4 For handing in on 4 December 2009

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1 State and prove a version of Iwasawa's Lemma in which 'abelian' is replaced by 'soluble'.

2 (a) How many Sylow 3-subgroups does A_6 have? Show that A_6 acts 2-transitively on them, by conjugation.

(b) Apply Iwasawa's Lemma to this action to show that A_6 is simple.

3 Compute the addition and multiplication tables for the fields

(a) $\mathbb{F}_4 = \mathbb{F}_2[x]/(x^2+x+1)$;

(b) $\mathbb{F}_8 = \mathbb{F}_2[x]/(x^3+x+1)$;

(c) $\mathbb{F}_9 = \mathbb{F}_3[x]/(x^2+1)$.

4 Let $G = \mathrm{GL}_n(q)$. Prove that $Z(G) = \{\lambda I_n \mid 0 \neq \lambda \in \mathbb{F}_q\}$, where I_n is the $n \times n$ identity matrix.

5 How many k-dimensional subspaces are there in a vector space of dimension n over the field of q elements?

6 Prove, by induction on n or otherwise, that the characteristic polynomial of the matrix

$$\begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & \\ \vdots & & \ddots & \ddots & \\ 0 & & & & 1 \\ a_0 & a_1 & a_2 & \dots & a_{n-1} \end{pmatrix}$$

is $x^n - a_{n-1}x^{n-1} - \cdots - a_1 x - a_0$.