

- 1** Use Sylow's theorems to prove that every group of order 40 has a proper non-trivial normal subgroup.  
Deduce that  $G$  is soluble.
- 2** Do the same for groups of order 80.
- 3** Do the same for groups of order 84.
- 4** Show that any group of order  $p^2q$ , where  $p$  and  $q$  are distinct primes, has a normal Sylow subgroup.
- 5** (Hard) Classify the groups of order  $pqr$ , where  $p, q, r$  are distinct primes (say  $p > q > r$ ).
- 6** Write down all the elements of  $\text{Aut}(C_2 \times C_2)$ . To which well-known group is it isomorphic?
- 7** Calculate  $\text{Inn}(G)$  when  $G = D_8$ . Show that  $\text{Aut}(G) \cong D_8$ .