

- 1** How many distinct composition series does the dihedral group  $D_8$  of order 8 have? Write them all down.
- 2**
- (a) Prove that the cyclic group  $C_n$  has a unique subgroup of each order  $d$ , where  $d|n$ .
  - (b) Write down all possible composition series for  $C_{60}$ .
  - (c) If  $n = p_1^{a_1} \cdot p_2^{a_2} \cdot \dots \cdot p_r^{a_r}$ , where  $p_1, \dots, p_r$  are distinct primes, how many distinct composition series does  $C_n$  have?
  - (d) (Harder.) How many distinct composition series does the dihedral group  $D_{2n}$  have?
- 3** Let  $G$  be the subgroup of  $S_7$  generated by the permutations  $(1, 2, 3, 4, 5, 6, 7)$  and  $(2, 6, 5, 7, 3, 4)$ .
- (a) Find a normal subgroup  $N$  of  $G$ . To what well-known group is  $G/N$  isomorphic?
  - (b) Hence or otherwise find a composition series for  $G$ .
  - (c) Show that  $G$  has exactly seven conjugacy classes, and write down representatives for each of them.