The four-colour theorem

MSc/MSci/BSc project suggestion

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Pre/co-requisites: Some familiarity with combinatorics and graph theory would be an advantage.

Description: The first record of the four-colour conjecture is in a letter written in 1852. The conjecture states that every 'regular' map can be coloured with just four colours, so that adjacent regions are coloured different colours. A 'proof' appeared in print in 1879, but in 1890 was shown to be fallacious. The problem remained open until 1976, when Appel and Haken published a very long account of the proof, which was mainly carried out by a computer checking thousands of cases.

Possible ways to focus the project are (a) look at the techniques of proof, especially the ideas of unavoidability and reducibility, (b) explore related colouring problems, such as edge-colouring of graphs, (c) colouring maps on surfaces of higher genus.

References: A great deal has been written on this subject, and you will need to be selective in designing a project around it. Useful sources include the following books:

1) R. Fritsch and G. Fritsch, The four-colour theorem: history, topological foundations, and idea of proof, Springer, 1998.

2) O. Ore, The four-color problem, Academic Press, 1967.

3) T. L. Saaty and P. C. Kainen, The four-color problem: assaults and conquest, McGraw–Hill, 1977, Dover reprint, 1986.

4) R. A. Wilson, Graphs, colourings and the four-colour theorem, OUP, 2002.

5) R. J. Wilson, Four colours suffice: how the map problem was solved, Penguin, 2002.