Netting the Monster

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The Monster is the largest of the 26 so-called ‘sporadic’ simple groups, and has nearly 10^{54} elements. The smallest conjugacy class is a class of around 10^{20} elements of order 2, known as 6-transpositions because the product of any two of them has order at most 6. These 6-transpositions form the basis of much recent work on the Monster and related structures such as the Griess algebra, the ‘Moonshine’ vertex operator algebra, Matsuo algebras and so on. In some sense, the study of such (commutative, non-associative) algebras was initiated by Simon Norton some 40 years ago, and he came up with the idea of ‘braiding’ the 6-transpositions into ‘nets’.

Given a triple (a, b, c) of 6-transpositions, you can replace it by a triple (b, a^b, c) with the same product abc. Repeating this operation you get back to where you started in at most 6 steps, depending on the order of ab. There are similar operations braiding a, c or b, c instead of a, b. You can therefore construct a polyhedron (or ‘net’) with vertices of degree 3, corresponding to the triples (a, b, c), and faces of degree at most 6. If all faces are hexagons, the polyhedron is topologically a torus, otherwise it is a sphere.

The problem is to classify the conjugacy classes of these nets. There are more than 13575 classes of nets (the precise number is not known), and they can be partially classified by (1) the conjugacy class of the element abc, and (2) the conjugacy class of the subgroup generated by a, b, c. In his 2005 PhD thesis, Richard Barraclough produced a complete list of nets for which $\langle a, b, c\rangle$ commutes with an element of prime order p, when $p \geq 5$.

The most interesting, and difficult, case, however, is when $\langle a, b, c\rangle$ has trivial centralizer, especially if a, b, c generate the Monster itself. The project would be to extend Barraclough's results as far as possible in this direction. It will require both theoretical and computational work. Other projects in group theory are also available.