

Key Objectives

In order to be reasonably sure of passing the examination in “Topics in Probability and Stochastic Processes” with a reasonable grade you should:

- Be familiar with the basic notions of probability including: definition of a stochastic process, distributions of discrete and continuous random variables, calculation of expectation values, convolution formula for distribution of a sum.
- Know the definition of a **renewal process** and the associated random variables. Be able to derive the expression for the renewal function in terms of the distribution of interoccurrence times; perform explicit calculations for simple cases such as (i) the Poisson process and (ii) a gamma density of interoccurrence times.
- Know and apply key results about the asymptotic behaviour of renewal processes including: the elementary renewal theorem and its derivation from the Blackwell renewal theorem; limiting distributions of number of renewal events, age, and excess life.
- Understand what is meant by a delayed renewal process and a stationary renewal process. Be prepared to analyse scenarios involving these and other simple generalizations such as cumulative (renewal-reward) processes.
- In the case of a discrete renewal process, be able to use the renewal argument, for example, to derive the equation satisfied by the renewal function. Understand the form and solution of the general renewal equation.
- Know the definition of a **continuous-time Markov chain** and the properties of the transition probability matrix. Understand the corresponding descriptions in terms of the infinitesimal matrix (generator) and the sojourn times in the embedded discrete-time Markov chain.
- Understand the arguments leading to forward and backward equations; be able to calculate the transition probability matrices corresponding to simple generators.
- Know the ergodic theorem for continuous-time Markov chains and be able to find the stationary distributions for simple examples.
- Know the definition of **Brownian Motion**. Be able to calculate the means and covariance functions of related stochastic processes and determine whether, or not, they are standard Brownian motions.
- Understand the reflection principle and be able to use it to calculate the time to first reach a level and the zeros of Brownian motion.

The exam will have duration 3 hours and the rubric will state “You should attempt all questions. Marks awarded are shown next to the questions.”