MTH5119 Test 2009 Solutions

Q1. Under SRS size n for any population size $N(>n)$

(2) $E[\bar{y}] = \bar{Y}$ [ie the sample mean is an UE of the population mean]

(3) $Var[\bar{y}] = \frac{1-f}{n} S^2$ [where $f = n/N$ is the sampling fraction]

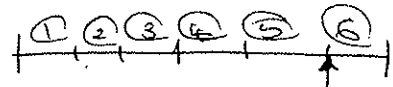
(2) $E[s^2] = S^2$ [ie the sample variance is an UE of the pop. variance]

(3) Key step in proof: Write $\bar{y} = \frac{1}{n}\sum_{i=1}^{N} t_i Y_i$ where $t_i = 1$ if unit i is in the sample (zero otherwise) [Sample indicator r.v.]

Q2. Initial sample 1 2 3 4 5 6 $n=6$

$k = n+1 = 7$: Select unit 7 with prob. $[n/k+1] = 6/7$

$u_1 = .4\ldots < 6/7$ so select unit 7 and replace



$u_2 = .8332 < 5/6$ (7 4/6) so replace unit (5) sample now 1 2 3 4 6 7

Select unit 8 with prob. $6/8$, $u_3 = .5\ldots < 6/8$ so select unit 8 and replace ~~1, 2, 3, 4, 6, 7~~? $u_4 = .0\ldots < 1/6$ so replace (1) sample now 2 3 4 6 7 8. Select unit 9 with prob. $6/9 = 2/3$

$u_5 = .7\ldots > 6/9$ so do not select unit 9.

Final sample 2 3 4 6 7 8

[3/10 One mark deducted for each error, 6 only/10 if appropriately self consistent, 5/10 for some idea, 5/10 if used 1 digit every time, 7/10 if used 2 or 3 digits every time]

Q3. (4) post-stratified estimate is $\bar{y}_{st} = \sum_h W_h \bar{y}_h = \frac{325}{725} \times 15.67 + \frac{400}{725} \times 8.89 = \underset{(4\,SF)}{11.93}$

(6) $v[\bar{y}_{st}] = \sum_h W_h^2 \frac{1-f_h}{n_h} s_h^2 = \left(\frac{325}{725}\right)^2 \left(\frac{1}{10} - \frac{1}{325}\right)(3.65)^2 + \left(\frac{400}{725}\right)^2 \left(\frac{1}{12} - \frac{1}{400}\right)(2.08)^2$

$= 0.3659$ (3) [s.e. $= 0.6049$] 4 s.F.

Q4 (2) $\hat{R} = \frac{\bar{y}}{\bar{m}} = \frac{50}{10} = \mathbf{5}$

(1) $s_y^2 = [634 - 50^2/5]/4 = 33.5$

(1) $s_{ym} = [127 - 5\times10\times10/5]/4 = 6.75$

(1) $s_m^2 = [26 - 10\times10/5]/4 = 1.5$

$v[\hat{R}] = \frac{1-f}{n\bar{m}^2}\left(s_y^2 - 2\hat{R}s_{ym} + \hat{R}^2 s_m^2\right) \approx \underset{(1)}{\frac{1-0}{5\times4}}$ (For neglecting fpc as v. large pop^n) $(33.5 - 10\times6.75 + 25\times1.5)$

$= 3.5/20 = \underset{(3)}{0.175}$ (2)