MTH5119 Test Solutions & Marking Scheme

Q1 $MSE[e] = Var[e] + (E[e] - \theta)^2$ (Lose 1 for just Var + bias²)

Q2 3 Sample values & labels

		$\bar{y}$	s^2	prob.
1, 2	1 2	1.5	0.5	1/3
1 3	1 6	3.5	12.5	1/3
2 3	2 6	4.0	8.0	1/3
	MEAN	(3)	(7)	

(one of these lose 1/2 if not) (Lose 1 if no probs)

$\bar{y}$ is unbiased as $\bar{Y} = \frac{1+2+6}{3} = 3$ (1)

s^2 is unbiased as $E[s^2] = S^2 = 7$ (1)

$Var[\bar{y}] = E[\bar{y}^2] - \theta^2 = \frac{1}{3}(1.5^2 + 3.5^2 + 4^2) - 9 = \frac{7}{6}(1.16) = \frac{1-f}{n}s^2$ as $\frac{1-\frac{2}{3}}{2} = \frac{1}{6}$ (1)

Q4. Initial sample {1, 2} Select new unit 3 with prob 2/3 $u_1 = .68\ldots > 2/3$ so do not select, Select new unit 4 with prob. $2/4 = 0.5$ $u_2 = .5\ldots > .5$ so do not select

Select new unit 5 with prob $2/5 = 0.4$ $u_3 = .0\ldots < 0.4$ so select new unit 5.

Replace 1 ① 2 ② using $u_4 = .5 > 0.5$ so replace unit ②.

Final sample is {1, 5}

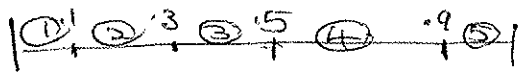
Q5. Sample values 0, 0, 1, 3, 5 $\rightarrow \bar{y}' = 9/5 = 1.8$ (1) $s^2 = \frac{1}{4}(5^2 + 3^2 + 1^2 - 9^2/5) = \frac{94}{20} = 4.7$ (1)

so $\hat{T}_c = N\bar{y}' = 180$ (1) (ft) $v[\hat{T}] = N^2\frac{(1-f)}{n}s^2 = 10000(1 - \frac{5}{100}) \times 4.7 = 8930$ (1) (ft)

Q6 $A = 75$. $\bar{y}_c = (1+3+5)/3 = 3$ $s_c^2 = \frac{1}{2}(2^2 + 0^2 + 2^2) = 4$ (1)

$\tilde{T}_c = 75 \times 3 = 225$ (1) $v[\tilde{T}_c] = \frac{A^2}{a}(1 - a/A)s_c^2 = \frac{75^2}{3}(1 - \frac{3}{75}) \times 4 = 7200$ (1)

Q7



$u_1 = .6\ldots \rightarrow$ select unit (4)

$u_2 = .8\ldots \rightarrow$ select unit (4)

variance estimate is zero