Queen Mary
University of London

B. Sc. Examination by course unit 2010

~~December 17th/18th~~

MTH 5119 Sampling, Surveys and Simulation: Coursework 4

~~(to hand back in presentation)~~

Date and time: December 1st 2009, 2pm

Hand in (GREEN box, ground floor) by 1145 Friday 11th December. This counts 4% of the overall assessment.

Question 1 In Lab 5,

(a) what is the variance of the unbiased estimator under PPS with replacement? [1] 69.5115

(b) give the estimated coverage of this estimator and a 95% C.I. for it. [2] 84.89% (0.8419, 0.8559)

(c) How does this strategy compare in terms of precision with the two under SRS ($n = 4$) using the ratio and sample mean estimators? [2]

worse than the ratio estimator, but better than the sample mean

Question 2 In Lab 6, what are your conclusions from the simulation results in

(a) step 2 (Pearson's X^2)? no evidence against the hypothesis of [2] $U(0,1)$

(b) step 3 (Anderson-Darling (AD))? no evidence against the hypothesis of [2] exponential mean 2

(c) step 4 (Ryan-Joiner/Shapiro-Wilk)? [2]

no evidence against the hypotheses of $N(0,1)$ for (i) Y (ii) Z

(d) step 4 (scatterplot of Y vs. Z)? [2]

no evidence against the hypothesis of independence between Y and Z

Question 3 In Lab 7,

(a) (step 1) What does your sequence of sample means demonstrate? [1]

The Law of Large Numbers, which (here) shows that the sequence converges to $\mu = 2$

(b) (step 2) What is the formula for μ, the mean of a Gamma distribution, if it has shape parameter α and scale parameter λ^{-1}? $\mu = \alpha/\lambda$ [1]

(c) (step 3) What are your conclusions from the observed value of the Ryan-Joiner (Shapiro-Wilk) statistic? Evidence against the hypothesis of $N(0,1)$ [1]

(d) (step 4) What are your conclusions from the observed value of the AD statistic? [1]

No evidence against the hypothesis of Gamma(16, 0.125) (shape, scale) (mean = 2)

(e) (step 5) What are your conclusions from the observed value of the Ryan-Joiner (Shapiro-Wilk) statistic? [2]

evidence against the hypothesis of $N(0,1)$

(f) (step 7) Submit the two plots. What are your conclusions? [4]

Some evidence against the hypothesis of t_4 but no evidence against t_3

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TURN OVER

$$\frac{1}{\beta-\alpha} = \frac{1}{2/3-1/2} = 6$$

Question 4 In Lab 8,

(a) What is the mean waiting time in a single server queueing system with independent exponentially distributed interarrival times, mean $\alpha^{-1} = 2$) and independent exponentially distributed service times, mean $\beta^{-1} = 1.5$? [1]

6.27544 (any answer around 6 acceptable)

(b) What is your simulation estimate of the above quantity? Why is it different? [2] simulation error (plus some small burn-in effect)

(c) What is the mean waiting time under the appointments system? [1]

5.59384 (any answer around 5 acceptable)

(d) Order (smallest first) the mean waiting times in the FOUR cases of single server/two servers, and Poisson arrivals/Appointments with normal error. [3]

appointments two servers, Poisson arrivals two servers, appointments single server, Poisson arrivals single server.

Question 5 (a) Show that if U is a uniform random variable on the interval $(0, 1)$ then $X = -\frac{1}{\lambda}\log U$ has an exponential distribution with parameter $\lambda > 0$. [4]

$x = -\frac{1}{\lambda}\log u \Leftrightarrow u = e^{-\lambda x}$ is 1,1 from $(0,1)$ to $(0,\infty)$

$\frac{du}{dx} = -\lambda e^{-\lambda x}$ so $f_X(x) = f_U(e^{-\lambda x})\left|\frac{du}{dx}\right|$
$= \lambda e^{-\lambda x}$ for any $0<x<\infty$ as $f_U(u)=1$

(b) Using the random string

756260135,

simulate two values from an exponential distribution mean 2, to 4 s.f. [3]

$\lambda = 0.5$ so $u_1 = 0.7562$, $u_2 = 0.6013 \Rightarrow x_1 = 0.5589$, $x_2 = 1.576$

(c) Hence generate the values of a pair of independent standard normal random variables. Take x_1 and u_2 say (x_2 and u_1 also OK) [5] to give

$y = \sqrt{x_1}\cos(2\pi u_2)$, $z = \sqrt{x_1}\sin(2\pi u_2) \longrightarrow y = -0.8042 \times 0.7476 = -0.6012$
$z = -0.5944 \times 0.7476 = -0.4444$

Question 6 (a) If $Y \sim U(-\pi/2, +\pi/2)$ with pdf

$$f_Y(y) = 1/\pi, \qquad -\pi/2 < y < +\pi/2, \text{ zero otherwise,}$$

$x = \tan y \Leftrightarrow y = \tan^{-1} x$ is 1,1 from $(-\pi/2, \pi/2)$ to $(-\infty,\infty)$

show that $X = \tan Y$ has pdf

and $f_X(x) = f_Y(\tan^{-1}(x)) \cdot \left|\frac{dy}{dx}\right| = \frac{1}{\pi(1+x^2)}$ and $\frac{dy}{dx} = \frac{1}{1+x^2}$, $f_Y(y) = 1/\pi \quad -\pi/2<y<\pi/2$

$$f_X(x) = \frac{1}{\pi(1+x^2)}, \qquad -\infty < x < +\infty, \text{ zero otherwise,}$$

called the *Cauchy* distribution. [4]

(b) What is the median of X? m s.t. $\int_{-\infty}^{m} \frac{1}{\pi(1+x^2)} = \frac{1}{2}$, by symmetry about 0 m must be 0 [2]

(c) Does X have finite mean? [2]

No, as $\int_{-\infty}^{\infty} \frac{x}{\pi(1+x^2)} = \left[\frac{1}{2\pi}\log(1+x^2)\right]_{-\infty}^{\infty}$ does not exist

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End of Paper

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