Queen Mary
University of London

B. Sc. Examination by course unit 2010

MTH 5119 Sampling, Surveys and Simulation: Coursework 4

Date and time: December 1st 2009, 2pm

Hand in (GREEN box, ground floor) by 1145 Friday 11th December. This counts 4% of the overall assessment.

Question 1 In Lab 5,

(a) what is the variance of the unbiased estimator under PPS with replacement? [1]

(b) give the estimated coverage of this estimator and a 95% C.I. for it. [2]

(c) How does this strategy compare in terms of precision with the two under SRS ($n = 4$) using the ratio and sample mean estimators? [2]

Question 2 In Lab 6, what are your conclusions from the simulation results in

(a) step 2 (Pearson's X^2)? [2]

(b) step 3 (Anderson-Darling (AD))? [2]

(c) step 4 (Ryan-Joiner/Shapiro-Wilk)? [2]

(d) step 4 (scatterplot of Y vs. Z)? [2]

Question 3 In Lab 7,

(a) (step 1) What does your sequence of sample means demonstrate? [1]

(b) (step 2) What is the formula for μ, the mean of a Gamma distribution, if it has shape parameter α and scale parameter λ^{-1}? [1]

(c) (step 3) What are your conclusions from the observed value of the Ryan-Joiner (Shapiro-Wilk) statistic? [1]

(d) (step 4) What are your conclusions from the observed value of the AD statistic? [1]

(e) (step 5) What are your conclusions from the observed value of the Ryan-Joiner (Shapiro-Wilk) statistic? [2]

(f) (step 7) Submit the two plots. What are your conclusions? [4]

Question 4 In Lab 8,

(a) What is the mean waiting time in a single server queueing system with independent exponentially distributed interarrival times, mean $\alpha^{-1} = 2$) and independent exponentially distributed service times, mean $\beta^{-1} = 1.5$? [1]

(b) What is your simulation estimate of the above quantity? Why is it different? [2]

(c) What is the mean waiting time under the appointments system? [1]

(d) Order (smallest first) the mean waiting times in the FOUR cases of single server/two servers, and Poisson arrivals/Appointments with normal error. [3]

Question 5 (a) Show that if U is a uniform random variable on the interval $(0, 1)$ then $X = -\frac{1}{\lambda}\log U$ has an exponential distribution with parameter $\lambda > 0$. [4]

(b) Using the random string

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simulate two values from an exponential distribution mean 2 to 4 s.f. [3]

(c) Hence generate the values of a pair of independent standard normal random variables. [5]

Question 6 (a) If $Y \sim U(-\pi/2, +\pi/2)$ with pdf

$$f_Y(y) = 1/\pi, \qquad -\pi/2 < y < +\pi/2, \text{ zero otherwise},$$

show that $X = \tan Y$ has pdf

$$f_X(x) = \frac{1}{\pi(1 + x^2)}, \qquad -\infty < x < +\infty, \text{ zero otherwise},$$

called the *Cauchy* distribution. [4]

(b) What is the median of X? [2]

(c) Does X have finite mean? [2]

End of Paper