MTH4106 **Introduction to Statistics**

Test 1 **24 February 2011, 1410–1450**

Write your name and student number in the spaces below.

Answer all questions. Write all your answers in the boxes provided.

Name: _____

Student Number: _____

Electronic calculators may be used. Please state here the name and type of machine used.

Note that there is an extract from the *New Cambridge Statistical Tables* on the last page of this test.

1 (20 marks) Australian biologists caught 15 pigeons in the desert. Here is the Minitab output showing the weight (in grams) of the dry seeds in their stomachs.

```
Stem-and-leaf of dryseed   N = 15
Leaf Unit = 0.1

 2   0  23
 6   0  5588
(3)  1  344
 6   1  56
 4   2  1
 3   2  5
 2   3  0
 1   3  8
```

Give the five-number summary for these data.

2 (20 marks) Let X be a discrete random variable all of whose values are non-negative integers.

(a) Define the *probability generating function* of X.

(b) State a result which gives $\mathbb{E}(X)$ in terms of the probability generating function of X.

(c) Prove the result stated above.

3 (20 marks) Let p be the proportion of registered UK voters who think that it is better to increase income tax than to increase university students' fees. A politician wants to estimate p, so he will take a random sample of 200 voters, with replacement, and ask them "is it better to increase income tax than to increase university students' fees?" Let X be the number who answer "yes", and let $Y = X/200$. The politician will use Y as an estimator for p.

(a) State the distribution of X.

(b) Find the bias of Y.

(c) What would be different if the sample were taken without replacement?

4 (20 marks) Let U be a continuous random variable which is uniform on the interval $[0, 1]$, and put $Y = U^2$.

(a) Write down the cumulative distribution function of U.

(b) Write down the probability density function of U.

(c) Find the cumulative distribution function of Y.

(d) Find the median of Y.

5 (20 marks) Let p be the proportion of meat-eaters among students at Queen Mary. The catering manager thinks that if $p < 0.3$ then she should withdraw meat from the menu and increase the number of vegetarian options. Her null hypothesis is $\mathrm{H}_0 : p \geqslant 0.3$ and her alternative hypothesis is $\mathrm{H}_1 : p < 0.3$.

With the help of a professor of statistics, she is going to take a random sample of 12 students and record the number X who say that they are meat-eaters. She will use X as the test statistic, with rejection region $\{0, 1\}$.

(a) What is the significance level of this test?

(b) She asks 12 students, and finds that exactly three of them are meat-eaters. As a statistician, what do you conclude?