QUEEN MARY, UNIVERSITY OF LONDON

MTH 4106 Introduction to Statistics

Practical 9 13 March 2012

Today we will

- draw graphs to show how well the normal distribution approximates the binomial and Poisson distributions,
- use an example to demonstrate the Law of Large Numbers.

You will be asked to include some of these graphs in the Feedback question on the next assignment.

1 (Normal approximation to a symmetric Binomial distribution) If $X \sim \mathrm{Bin}(9, 0.5)$ then X has mean 4.5, variance 2.25 and standard deviation 1.5. So it should be well approximated by a normal random variable Y with mean 4.5 and standard deviation 1.5. We shall plot both of these on the same graph. Use

`Graph` $\rightarrow$ `Probability Distribution Plot ...`,

choose `Two Distributions` and click `OK`. Make the first distribution $\mathrm{Bin}(9, 0.5)$ and the second distribution $N(4.5, 2.25)$, and choose to overlay them on the same graph. You should get a much prettier version of the hand-drawn figure that I showed in lectures.

Paste the graph into the **Report Pad** and write a few lines explaining what it shows.

2 (Normal approximation to a non-symmetric Binomial distribution) Now consider the $\mathrm{Bin}(24, 0.4)$ distribution. Find out which normal distribution has the same mean and variance as this.

Plot its probability density function on the same graph as a histogram of the binomial probability mass function. Annotate this graph with a short explanation.

3 (Normal approximation to a Poisson distribution)
Let $X \sim$ Poisson(16). We know that X is well approximated by a normal distribution. Let Y be the normal random variable which best approximates X. What are the mean and variance of Y?

Plot the probability mass function of X and the probability density function of Y on the same graph. Copy it to the **Report Pad** and add a few lines of explanation and comment.

4 (Fresh Start) Save and close the **Report Pad** before starting the next part of the practical. You may prefer to `Exit` from Minitab and start again.

5 (Simulation to illustrate the Law of Large Numbers) Put the numbers 1, ..., 1000 into the first 1000 rows of column `C1`. We will regard the number in row n of `C1` as being n. You may find it helpful to give each column suitable name when you create it.

You are going to simulate a distribution of your choice. Choose any distribution that we have discussed in *Introduction to Statistics*, and write down its name, expectation and variance.

Start your new **Report Pad** by explaining your chosen distribution, and giving its expectation and variance.

Simulate 1000 rows of data from your chosen distribution and store them in column `C2`. (See Practical 8 for how to do this.) We will think of these as x_1, ..., x_{1000}.

For $n = 1, \ldots, 1000$, calculate x_n^2 and store it in row n of column `C3`.

Under `Calc` → `Calculator`, find the function `Partial sum` and apply it to `C2`, storing the result in `C4`. Verify that this puts $\sum_{i=1}^{n} x_i$ into row n of column `C4`.

The mean of the first n rows of C2 is $\left(\sum_{i=1}^{n} x_i\right)/n$. Put this into row n of column C5, by dividing C4 by C1.

Plot C5 against C1, adding a suitable title. What does this graph show?

Copy the graph into a report pad and make brief comments on it. Then delete the graph.

Use C6 to store the partial sums $\sum_{i=1}^{n} x_i^2$. Then use C7 to store the sample variance

$$\left[\sum_{i=1}^{n} x_i^2 - n\left(\sum_{i=1}^{n} x_i/n\right)^2\right] \Big/ (n-1).$$

You will get a warning message to show that this calculation is not valid in row 1. Press Cancel, and the rest of the column will be calculated.

Then calculate C8 as $\sqrt{\texttt{C7/C1}}$. What is this quantity?

Plot C8 against C1, giving the graph a suitable title. Paste the graph into the report pad.

Edit the report to explain briefly what you have done and what the two graphs show.