

QUEEN MARY, UNIVERSITY OF LONDON

MTH 4106

Introduction to Statistics

Assignment 8

For handing in on 13 March 2012

You should attempt all of these questions, as they are designed to help you to learn and understand the material in the course.

The 'Feedback' question is the one for handing in. Write your name, student number and group number at the top of your answer before handing it in. Staple all the pages together. Hand it to your allocated tutor when (s)he asks for it during your allocated Minitab laboratory session on Tuesday 13 March 2012.

If you want help on any of the other questions, or want to check that you have done them correctly, you may ask any tutor during your laboratory session or ask me in any of my office hours.

1 (Your chance to mark the mid-term test) Here are some things that were written as parts of answers to the mid-term test. Explain what is wrong with each of them.

(a) Minimum $\Rightarrow 3$.

(b) $Q_1 = \frac{n}{4} = \frac{26}{4} = [6.5] = 7^{\text{th}}$ value.

(c) $G(x) = \sum_i p_i x^i \Big|_{i=1}$

(d) $G(x) = \sum_i p_i t^i$.

(e) If $p = \frac{1}{4}$ then $p_i = \frac{1}{4^i}$.

(f) $X \sim \text{Geom}(p) = \frac{pt}{1 - qt}$.

(g) $\frac{\frac{1}{4}t}{1 - \frac{3}{4}t} = \frac{\frac{1}{4}t}{\frac{1}{4}t} = 1$.

(h) $\text{MSE}(T) = [\mathbb{E}(T - p)]^2$.

(i) $f(x) = \begin{cases} \frac{1}{3} & \text{if } x \in \mathbb{R} \\ 0 & \text{otherwise} \end{cases}$

(j) The pdf is 16.

(k) The pdf is $f_x(Y)$.

(l) $3e^{-3\left(\frac{y-1}{5}\right)} = 3e^{\frac{-3y+3}{-15}}$.

2 (a) Use the random number button on your calculator to generate 10 realizations of the random variable U which is uniform on $[0, 1]$.

(b) Use these 10 random numbers to

(i) simulate 10 values from the $\text{Bin}(16, 0.3)$ distribution;

(ii) simulate 10 values from the $\text{Exp}(3)$ distribution.

3 Let X and Y be independent random variables such that $X \sim N(1, 4)$ and $Y \sim N(-3, 9)$. Find

(a) the joint probability density function of (X, Y) ;

(b) $\text{Var}(X - Y)$;

(c) $\mathbb{E}(X^2)$;

(d) $\mathbb{E}(X + Y)$;

(e) $\mathbb{P}(2X < Y + 6)$;

(f) the probability density function of $2X + Y$.

4 The probability that a light bulb will fail in a year is 0.75, and light bulbs fail independently. If 192 bulbs are installed, what is the probability that the number which fail in a year lies between 140 and 150 inclusive? Give the answer to four decimal places.

5 (Feedback) There are 300 students taking *MTH4106 Introduction to Statistics*. On the first day of March, each of them goes to the Student Union shop and buys a piece of fruit. For each of them independently, the probability that they buy an orange is $1/4$. Let X be the number of students taking *MTH4106 Introduction to Statistics* who buy an orange.

(a) Find $\mathbb{E}(X)$ and $\text{Var}(X)$.

(b) Find $\mathbb{P}(60 \leq X \leq 80)$. Give the answer to four decimal places.