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MAS 314 Design of Experiments

Practical 9 8 March 2007

In this practical we do some power calculations for an experiment on people where there are only two treatments.

Suppose that we wish to detect a difference of magnitude δ between the responses to the two treatments, and that we guess that the variance of each response is σ^2. If we use equal replication r for both treatments, then the variance of the estimator of the difference between the two treatments is

$$\frac{2}{r}\,\sigma^2$$

if we use a completely randomized design, an orthogonal block design, or a cross-over design. If we perform a significance test at level 5% and want to have probability at least 90% of detecting a difference as large as δ, then we saw in lectures that we need

$$(a+b)^2\,\frac{2}{r} \leq \left(\frac{\delta}{\sigma}\right)^2,$$

where a and b are the 97.5% and 90% points of the t distribution on d degrees of freedom, and d is the number of degrees of freedom for residual. We can rewrite this as

$$c \leq r, \tag{1}$$

where

$$c = \frac{2(a+b)^2}{(\delta/\sigma)^2}. \tag{2}$$

1 (Completelely randomized design) First we suppose that we use a completely randomized design using $2r$ people. Write down the formula for d in terms of r.

We shall do the calculations iteratively, starting $r = 3$. In that case, $d = 4$. Each step will correspond to one row in the table at the bottom of this page.

We begin by calculating a and b when $d = 4$.

`Data` → `Calculations...`

Click on [Functions ...] and then choose/type

Function class:	Inverse Probability
Function:	t distribution
Argument 1:	0.975
Argument 2:	4

Save the result in a scalar called `a`, and print the result in the Output Window.

Calculate and print b similarly, using 0.9 as the first argument.

Supppse that $\sigma = 10$ and $\delta = 15$. Use Equation (2) to calculate c and print it in the Output Window. Write the current values of d and c in the first row of the table at the foot of the page.

Is Equation (1) satisfied? You should find that it is not. So start the next step by putting r equal to the smallest integer bigger than the current value of c. Write it in the second row of the table. Calculate the new value of d, and write it in the table too.

There is an easy way to calculate the new value of c. Open the Input Log. The last nine lines in this should show what you did to calculate and print a, b and c. Cut and paste these nine lines into the Input Window. The value 4 occurs at two places in these lines: edit the lines to replace both of these by the new value of d. Then highlight the nine lines, and then submit them. The new value of c should appear in the Output Window. Copy it into the table.

Continue in this way until Equation (1) is satisfied.

Step	r	d	c
1	3		
2			
3			

How many people are needed for the completely randomized design?

2 (Matched pairs design) Suppose that by putting people into homogeneous blocks of size two we can reduce σ to 9. Assume that we still use $2r$ people, in a complete-block design. Write down the new formula for d in terms of r.

Again, start with $r = 3$, and fill in the table below until Equation (1) is satisfied.

Step	r	d	c
1	3		
2			
3			

How many people are needed for the matched pairs design?

3 (Cross-over design) Now we consider a cross-over design. Why should r be even for such a design?

Suppose that the cross-over design reduces σ to 7.
Write down the new formula for d in terms of r.

This time, start with $r = 4$, and fill in the table below until Equation (1) is satisfied.

Step	r	d	c
1	4		
2			
3			

How many people are needed for the cross-over design?