Combinatorics: draft landscape document

Activity in combinatorics is difficult to judge by looking at output. Firstly, as Tim Gowers has cogently argued in the IMU-sponsored book *Mathematics: Frontiers and Perspectives* (AMS 1999), it is a good example of a subject where techniques may be more important than theorems. Second, a lot of combinatorics is done by people who do not label themselves as combinatorialists (algebraists, geometers, topologists, logicians) or even mathematicians (they may belong to departments of computer science, information security, or even management). It is not easy to be sure that all this activity has been noted.

On the other hand, the UK combinatorics community has done a lot to encourage the discipline, and their records show the activity in the subject. There is a biennial international conference, whose business meeting elects a committee to oversee this activity. (Records of past conferences are available on the web.) Other features include the *British Combinatorial Bulletin* published every year with details of people, courses, and publications; the committee web page; an annual postgraduate conference; and several regular one-day meetings at the Open University, UCL, and Reading. The survey papers of the nine invited speakers at each BCC are published in the LMS Lecture Note series by Cambridge University Press, and form a valuable resource. As evidence of the impact of the conference, the list of problems from the problem session was top of the list of most-downloaded papers from the journal *Discrete Mathematics* in January–March 2003.

In addition, the journal *Combinatorics, Probability and Computing* is published in the UK, and we have a good presence on the editorial boards of the top international journals in combinatorics.

All areas of combinatorics are studied in the U.K. There are particularly strong traditions in graph theory, design theory, Ramsey and extremal combinatorics, and symmetric functions (established in part by people such as C. St. J. A. Nash-Williams, R. A. Fisher, R. Rado and I. D. Macdonald), but there is also considerable expertise in probabilistic combinatorics, combinatorial number theory, asymptotic enumeration (especially related to algebra), matroids, finite fields and finite geometries, group actions and representation theory, symmetric functions and orthogonal polynomials, coding theory, information security, algorithmic combinatorics, and computational complexity. The Bulletin shows recent output of British combinatorialists, and archived versions give a longer view. However, these records are not complete, and internet searches for some leading British combinatorialists produce further material.

The recent Fields medals for two British researchers with at least one foot in the combinatorics camp (Gowers and Borcherds), and a meeting on "Combinatorics, Probability and Computing" at the Newton Institute, have helped to boost the subject. Gowers' work on Szemeredi's Lemma and the density version of van der Waerden's Theorem fall into *combinatorial number theory*, an area of strength which has also seen recent work by Green and Walters. Traditional areas of *extremal combinatorics* are also well represented with Brightwell, Hilton, Jackson, Leader, Scott, Thomason and Woodall active in this area. Cooper,

McDiarmid and Riordan are prominent in pure *probabilistic combinatorics*, as well as applications to telecommunications. A school of *combinatorial geometry* flourishes at University College, London, led by Barany and Larman. *Design theory* is strong, with Bailey, Cameron and Soicher at Queen Mary building links between combinatorics and statistics. The researchers at Royal Holloway are world leaders in combinatorial aspects of *information security*. Finally, there is some strong work on the boundary between symmetric structures in combinatorics, permutation groups, and model theory (Cameron, Ivanov, Liebeck, Macpherson, Neumann, Saxl).

However, the age profile in the subject gives serious cause for concern. There are plenty of older researchers with strong reputations, and a good number of Ph.D. students, but worryingly few postdocs and young researchers. As a result, recruitment to permanent positions in universities is difficult and the fields are distorted by large numbers of strong overseas candidates. In the past, Britain exported some very good researchers (P. D. Seymour a notable example), but this is not happening so much now.

The number of combinatorialists in computer science departments in the U.K. is rather low by international standards. An example of an area where there has been give-and-take between combinatorics, probability, and computer science is that of randomised algorithms; the UK presence in this area (at Edinburgh, Warwick, Leeds, LSE and other places) shows how things can be, but even here the lack of strong computer science input might mean missing opportunities raised by new topics such as the "Web graph".

The Web pages mentioned above are:

- British Combinatorial Bulletin,
  `http://www.cdam.lse.ac.uk/BCB/`
- British Combinatorial Committee,
  `http://www.maths.qmul.ac.uk/~pjc/bcc/`
- British Combinatorial Conferences archive,
  `http://www.maths.dundee.ac.uk/~kedwards/bcc/past.html`

Combinatorics is mostly done by individuals rather than groups. In some cases, there are organised "groups"; in others, I have simply mentioned a University department as if it were a group. Neither the list of groups in Appendix 1 nor the lists of members are intended to be complete. More information on departments can be found in the *British Combinatorial Bulletin*, List B.

Peter J. Cameron
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This document has been produced in consultation with the British Combinatorial Committee and other interested people.

Appendix 1: Research groups

Aberystwyth: McDonough, Mavron, Morris: designs, algebraic combinatorics
Birmingham: Bray, Butkovic, Curtis, Gardiner, Holmes, Parker, Wilson: representation theory, symmetric graphs, combinatorial optimization
Cambridge: Gowers, Leader, Saxl, Thomason, Bollobás, Green, Riordan: a wide range of topics in combinatorics
Dundee: Edwards: Graph colouring
East Anglia: Camina, Evans, Siemons: design theory, group actions
Edinburgh: Jerrum: Randomised algorithms, Markov methods
Exeter: Chapman, Vámos: Coding theory, algebraic combinatorics, matroids
Glamorgan: Perkins, Robertshaw, Rutherford, Smith: frequency assignment, codes, graphs, matroids
Glasgow: Anderson, Cohen, Hoggar, Spence: designs, graphs, finite fields
Imperial: Liebeck, James, Pretzel: representation theory, permutation groups
Kent: Clarke, Fleischmann, Hughes Jones, Pearce, Preece, Rees, Vowden, Woodcock: design theory, algebraic combinatorics
Leeds: Cooper, Dyer, Macpherson, Truss: model theory, approximation algorithms, homogeneous structures
Leicester: Marsh, Stewart, Thomas: model-theoretic complexity, combinatorial group theory, quantum groups
LSE: Alpern, Biggs, Brightwell, van den Heuvel, von Stengel: Algebraic, combinatorics, graph theory, operational research, game theory
Manchester/UMIST: Borovik, Booth, Ray, Rowley, Sandling, Walker: algebraic combinatorics, Coxeter matroids, combinatorial group theory
Newcastle: Dye, Duncan, King, Rees: finite geometry, combinatorial group theory, classical groups
Nottingham: Woodall: graph colouring
Open University: Grannell, Griggs, Holroyd, Quinn, Webb, Wilson: graph colouring, design theory, history of combinatorics
Oxford: Welsh, McDiarmid, Talbot: algorithmic combinatorics, complexity, probabilistic combinatorics, extremal combinatorics
Queen Mary: Bailey, Cameron, Jackson, Preece, Soicher, Stark: design theory, algebraic and probabilistic combinatorics, groups, graphs and matroids
Reading: Hilton: design theory, graph colouring
Royal Holloway: Blackburn, Burmester, Damerell, Essam, Martin, Mitchell, Murphy, Piper, Wild: information security, coding; design theory; percolation
St Andrews: Campbell, Constable, Robertson, Ruskuc: Combinatorial group and semigroup theory
Southampton: Jones, King, Lloyd, Potts, Singermann: groups and surfaces; enumerative combinatorics; optimization; history of combinatorics
Stirling: Bell, Rowlinson: Geometry of graph eigenvalues
UCL: Barany, Csornyei, Larman, McMullen, Preiss, Rogers, Scott: geometric combinatorics, graph theory
Warwick: Goldberg, Goldberg, Martin, Paterson: combinatorial and randomised algorithms

Appendix 2: Selected publications in 2002–3

P. P. Alejandro, R. A. Bailey and P. J. Cameron, Association schemes and permutation groups, *Discrete Mathematics* **266** (2003), 47–67.

N. L. Biggs, Chromatic polynomials and representations of the symmetric group, *Linear Algebra and its Applications* **356** (2002), 3–26.

B. Bollobás and G. R. Brightwell, The number of k-SAT functions, *Random Struct. Algorithms* **22** (2003), 227–247.

G. Brightwell and P. Winkler, A combinatorial approach to correlation inequalities, *Discrete Mathematics* **257** (2002), 311–327.

P. J. Cameron, Cycle index, weight enumerator and Tutte polynomial, *Electronic J. Combinatorics* **9(1)** (2002), #N2 (10pp).

D. Cvetkovic, P. Fowler, P. Rowlinson and D. Stevanovic, Constructing fullerene graphs from eignvalues and angles, *Linear Algebra and its Applications* **356** (2002), 37–56.

M. N. Ferencak and A. J. W. Hilton, Outline and nearly outline triple systems of even index, *Proc. London Mathematical Society* (3) **84** (2002), 1–34.

M. J. Grannell, T. S. Griggs and J. Siran, Recursive constructions for triangulations, *Journal of Graph Theory* **39** (2002), 87–107.

N. Hindman, I. Leader and D. Strauss, Image partition regular matrices—bounded solutions and preservation of largeness, *Discrete Mathematics* **242** (2002), 115–144.

M. W. Liebeck and A. Shalev, Bases of primitive permutation groups, pp. 147–154 in *Groups, Combinatorics and Geometry*, World Scientific, 2003.

B. Jackson and X. Yu, Hamilton cycles in plane triangulations, *Journal of Graph Theory* **41** (2002), 138–150.

M. R. Jerrum and J.-B. Son, Spectral gap and log-Sobolev constant for balanced matroids, FOCS'02, IEEE Computer Society Press, 2002, pp. 721–729.

A. V. Kostochka and D. R. Woodall, Total choosability of multicircuits, I, *Journal of Graph Theory* **40** (2002), 26–43.

S. A. Linton and R. Sebastiani (editors), Integration of Automated Reasoning and Computer Algebra Systems, a special issue of *Journal of Symbolic Computation* (4) **34** (2002).

C. McDiarmid, Concentration for independent permutations, *Combinatorics, Probability & Computing* **11** (2002), 163–178.

J. Oxley and D. Welsh, Chromatic, flow and reliability polynomials: The complexity of their coefficients, *Combinatorics, Probability & Computing* **11** (2002), 403–426.

O. Pikhurko and A. Thomason, Disjoint subgraphs of large maximum degree, *Discrete Mathematics* **248** (2002), 125–141.

D. Singerman, Unicellular dessins and a uniqueness theorem for Klein's surface of genus 3, *Bull. London Mathematical Society* **33** (2001), 701–710.

D. Stark, Information loss in top to random shuffling. *Combinatorics, Probability & Computing* **11** (2002), 607–627.

J. M. Talbot, Lagrangians of hypergraphs, *Combinatorics, Probability & Computing* **11** (2002), 199–216.