

A graph covering problem

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In 1983, Allan Schwenk posed a problem in the *American Mathematical Monthly* asking whether the edge set of the complete graph on ten vertices can be decomposed into three copies of the Petersen graph. He, and O. P. Lossers (the problem-solving group at Eindhoven University run by Jack van Lint – “oplossers” is Dutch for “solvers”) gave a negative solution in 1987. This year, Sebastian Cioabă and I considered the question: for which m is it possible to find $3m$ copies of the Petersen graph which cover the complete graph m times. We were able to show that this is possible for all natural numbers m except for $m = 1$. I will discuss the proof, which involves three parts: one uses linear algebra, one uses group theory, and one is bare-hands.

Of course this problem can be generalised to an arbitrary graph G :

Given a graph G on n vertices, for which integers m can one cover the edges of K_n m times by copies of G ?

I will say a bit about what we can do, and pose some very specific problems.