A graph covering problem
Peter Cameron

In 1983, Allan Schwenk posed a problem in the American Mathematical Monthly asking whether the edge set of the complete graph on ten vertices can be decomposed into three copies of the Petersen graph. He, and O. P. Lossers (the problem-solving group at Eindhoven University run by Jack van Lint – “oplossers” is Dutch for “solvers”) gave a negative solution in 1987. This year, Sebastian Cioabă and I considered the question: for which \( m \) is it possible to find \( 3m \) copies of the Petersen graph which cover the complete graph \( m \) times. We were able to show that this is possible for all natural numbers \( m \) except for \( m = 1 \). I will discuss the proof, which involves three parts: one uses linear algebra, one uses group theory, and one is bare-hands.

Of course this problem can be generalised to an arbitrary graph \( G \):

Given a graph \( G \) on \( n \) vertices, for which integers \( m \) can one cover the edges of \( K_n \) \( m \) times by copies of \( G \)?

I will say a bit about what we can do, and pose some very specific problems.