Conference matrices

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A *conference matrix* is an $n \times n$ matrix C with zeros on the diagonal and entries ± 1 elsewhere which satisfies $CC^\top = nI$. Such a matrix has the maximum possible determinant given that its diagonal entries are zero and the other entries have modulus at most 1.

Conference matrices first arose in the 1950s in connection with conference telephony, and more recently have had applications in design of experiments in statistics. They have close connections with other kinds of combinatorial structure such as strongly regular graphs and Hadamard matrices.

It is known that the order of a conference matrix must be even, and that it is equivalent to a symmetric matrix if $n \equiv 2 \pmod 4$ or to a skew-symmetric matrix if $n \equiv 0 \pmod 4$. In the second case, they are conjectured to exist for all admissible n, but there are some restrictions in the first case (for example, there no conference matrices of order 22 or 34). Statisticians are interested to know what is the maximum possible determinant in cases where a conference matrix does not exist.

I will give a gentle introduction to the subject, and raise a recent open question by Dennis Lin.