Abdullahi Umar has discovered that many celebrated sequences of combinatorial numbers, including the factorials, binomial coefficients, Bell, Catalan, Schröder, Stirling and Lah numbers solve counting problems in certain naturally defined inverse semigroups of partial bijections on a finite set. I will give an account of some of these results, together with the beginning of a study of $q$-analogues where we consider linear bijections between subspaces of a finite vector space (and some very interesting open problems arise).