The power graph of a finite magma

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A *magma* is simply a set with a binary operation (think of a group). The *directed power graph* of a magma has the elements of the magma as vertices, with an arc from x to y if y is a power of x; the *undirected power graph* has an edge joining two vertices if one is a power of the other. In order for this to make sense, we should probably require that our magma is *power-associative*, that is, a power of an element is independent of the bracketing used to calculate it.

Here is an example of a power-associative magma which is a loop (that is, it has identity and inverses) but not a group:

$*$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	0	4	2	5	3
2	2	4	0	5	3	1
3	3	2	5	0	1	4
4	4	5	3	1	0	2
5	5	3	1	4	2	0

Its power graph is a star.

Recently I showed that if two finite groups have isomorphic undirected power graphs, then they have isomorphic directed power graphs. I will talk about this result, and what the power graph of a group tells us about the group, and will speculate on extensions of the result to larger classes of power-associative magmas.