# Random mappings with exchangeable in–degrees

Jennie C. Hansen* and Jerzy Jaworski†

**Abstract**

A random mapping $T_n^{\hat{D}}$ maps the set $\{1, 2, ..., n\}$ into itself and it is constructed using a collection of exchangeable random variables $\hat{D}_1, ...., \hat{D}_n$ which satisfy $\sum_{i=1}^n \hat{D}_i = n$. In the random digraph, $G_n^{\hat{D}}$, which represents the mapping $T_n^{\hat{D}}$, the in-degree sequence for the vertices is given by the variables $\hat{D}_1, \hat{D}_2, ..., \hat{D}_n$ and, in some sense, $G_n^{\hat{D}}$ can be viewed as an analogue of the general independent degree models from random graph theory. We show that the distribution of many characteristics of the digraph, like the number of cyclic points, the number of components, the number of successors and predecessors of a typical vertex and the size of a typical component can be expressed in terms of expectations of various functions of $\hat{D}_1, \hat{D}_2, ..., \hat{D}_n$. We also consider two special examples of $T_n^{\hat{D}}$ which correspond to random mappings with preferential and anti-preferential attachment, respectively, and determine, for these examples, exact and asymptotic distributions for the statistics mentioned above. In this manner some known families of discrete probability distributions arise.

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*Actuarial Mathematics and Statistics Department and The Maxwell Insitute for Mathematical Sciences, Heriot–Watt University, Edinburgh EH14 4AS, UK. E-mail address: J.Hansen@ma.hw.ac.uk

†Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Umultowska 87, 61-614 Poznań, Poland. E-mail address: jaworski@amu.edu.pl