Acyclic orientations and poly-Bernoulli numbers

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Acyclic orientations of a graph arise in various applications, including heuristics for colouring. The number of acyclic orientations is an evaluation of the chromatic polynomial. Stanley gave a formula for the average number of acyclic orientations of graphs with n vertices and m edges. Recently we have found the graphs with the minimum number of acyclic orientations, but the more interesting question about the maximum number is still open.

The regular complete bipartite graph (on an even number of vertices) is thought to maximise the number of acyclic orientations. Unexpectedly, the number turns out to be a poly-Bernoulli number, one of a family of numbers connected with polylogarithms. We will try to explain these connections.