BCC Problem List

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This document contains the problems presented at problem sessions at the British Combinatorial Conference, from BCC12 to the present. All those problems which were published in the Conference proceedings, and some others, are included. I have annotated and updated them with further references wherever I know of any. Further information on any of the problems are very welcome, and will be included in subsequent versions of the problems. The proposers' addresses are the most recent known to me, but some are now long out of date; again, updates are welcome.

From BCC13 onwards, the problems (together with the contributed papers) have been published in the Research Problems section of *Discrete Mathematics*. Each problem appearing in this section receives a unique number, which I have given in the form **DMnnn**.

1 BCC12

BCC12 was held at the University of East Anglia, Norwich, 3–7 July 1989. Contributed papers were published in *Ars Combinatoria* **29** (1990); the problems were not included, but were circulated to participants.

Problem BCC12.1: Some doubly resolvable designs. Proposed by R. A. Bailey. Correspondent: R. A. Bailey.

For which n and k does there exist a doubly resolvable incomplete block design for nk treatments in n^2 blocks of size k, so that all concurrences are 0 or 1?

Such a design exists if there are k MOLS of order n. What about other values? In particular, does it exist for $(n,k) = (6,3)$, $(6,4)$, $(6,5)$ or $(10,3)$?

Editor's Note: These designs are now known as SOMAs (see Phillips and Wallis [88]). For recent results on their structure and classification, including existence for $(10,3)$, see Soicher [99]. Further information on semi-Latin squares is available from [9].

Problem BCC12.2: Latin squares with transitive groups. Proposed by R. A. Bailey. Correspondent: R. A. Bailey.

Do there exist any Latin squares, other than Cayley tables, whose automorphism groups are transitive?

Editor's note: The proposer answered her own question: there is such a square of order 6: see [7, p. 54]. A more difficult question would be to determine all such squares.

Problem BCC12.3: Two Diophantine equations. Proposed by Peter Cameron. Correspondent: Peter Cameron.

Let q,n,k be positive integers with $q > 1$, $n > 3$.

(a) Show that

$$\binom{k}{2} - 1 = \frac{q^n - 1}{q - 1}$$

has no solutions.

(b) Show that

$$\binom{k}{2} - 1 = q^n + 1$$

has only the solution $q^n = 64$, $k = 12$.

Remark: Richard Guy remarks that, for fixed n, these equations are presumably of the sort to which Faltings' theorem applies, in which case there are only finitely many solutions for each n. But surely these equations are less difficult than Fermat's last theorem!

Editor's Note: Hering [60] has obtained further results on these equations, including the fact that the second equation has only finitely many solutions if $q < 47$ or if n is divisible by 3.

Problem BCC12.4: Generalised quadrangles. Proposed by J. Tits and P. J. Cameron. Correspondent: Peter Cameron.

Show that there is no generalised quadrangle with parameters s, t, where s is finite and greater than 1, and t is infinite.

Note: A *generalised quadrangle* is a geometry of points and lines, with any two points on at most one line, such that if point P is not on line L, then P is collinear with a unique point of L. It has parameters s, t if each line contains $s+1$ points and each point lies on $t+1$ lines. If $s = 1$, it is a complete bipartite graph. The finiteness of t when $s = 2, 3$ was shown by Cameron and Kantor respectively.

Editor's Note: A simplified proof for the case $s = 3$ was given by Brouwer [18]. Finiteness in the case $s = 4$ was shown by Cherlin (unpublished), who gave a general method which can in principle deal with larger values of s (with ever-increasing amounts of hard labour).

Problem BCC12.5: Edge-density of infinite graphs. Proposed by Peter Cameron. Correspondent: Peter Cameron.

Let G be an infinite graph. List all *labelled* n-vertex subgraphs of G (a finite list, for each n), and let d_n be the average density of edges in the list (that is, the total number of edges divided by $\binom{n}{2}$ times the number of graphs in the list. Does $\lim_{n\to\infty} d_n$ exist?

Problem BCC12.6: Intersecting families. Proposed by P. J. Cameron, P. Frankl and W. M. Kantor. Correspondent: Peter Cameron.

Let $\mathcal{F} \subseteq \mathcal{P}(\{1, 2, \ldots, n\})$ be an intersecting family of sets which is maximal (that is, $|\mathcal{F}| = 2^{n-1}$) and regular (each point lies in the same number of elements of $\mathcal{F}$). Let m be the size of the smallest set in $\mathcal{F}$. It is known that $m \geq \frac{1}{2}\log n + c$; examples are known with $m \sim \sqrt{n}$. (See [28]). What is the truth?

Editor's Note: This problem has been solved by Aaron Meyerowitz [81], who showed that the lower bound is correct.

Problem BCC12.7: Forbidding divisibility. Proposed by P. J. Cameron and P. Erdős. Correspondent: Peter Cameron.

Let $f(n)$ be the number of sequences

$$1 \leq a_1 < \ldots < a_t \leq n$$

with $a_i \not| a_j$ for $i \neq j$. Prove that $\lim_{n\to\infty} f(n)^{1/n}$ exists.

Note: The limit should be about 1.58: see [27].

Problem BCC12.8: Inverting sign matrices. Proposed by Ömer Eğecioğlu. Correspondent: Ömer Eğecioğlu.

Consider two disjoint finite sets A and B, each partitioned into n^2 not necessarily nonempty subsets A_{ij} and B_{ij} for $i,j = 1,\ldots,n$, respectively. Suppose that we have a mapping $\text{sign} : A\cup B \to \{-1,1\}$. Let

$$a_{ij} = \sum_{x\in A_{ij}} \text{sign}(x), \qquad b_{ij} = \sum_{y\in B_{ij}} \text{sign}(y).$$

In many combinatorial situations, the numbers a_{ij} and b_{ij} are given not explicitly but *algorithmically*.

Put

$$I_{ij} = \bigcup_{k=1}^{n} A_{ik}\times B_{kj}, \qquad I = \bigcup_{i,j=1}^{n} I_{ij}.$$

Clearly $I \subseteq A\times B$. Denote by x_{ij} and u_{ij} generic elements of A_{ij}, and by y_{ij} and v_{ij} elements of B_{ij}. We turn I into a *signed space* by defining, for every $(x_{ik},y_{kj}) \in I$,

$$\text{sign}(x_{ik},y_{kj}) = \text{sign}(x_{ik})\,\text{sign}(y_{kj}).$$

Let $\mathbf{A} = ||a_{ij}||$, and $\mathbf{B} = ||b_{ij}||$, and suppose that $\mathbf{AB} = \mathbf{I}$. In many instances (for example, in the combinatorics of the transition matrices for symmetric function bases), one can provide a combinatorial proof of this fact (viz., $\mathbf{AB} = \mathbf{I}$) by means of a *sign-reversing involution* on I. That is, we can construct a permutation α on I satisfying

(i) $\alpha : I_{ij} \to I_{ij}$;

(ii) if $\alpha(x_{ik},y_{kj}) = (u_{il},v_{lj}) \neq (x_{ik},y_{kj})$, then $\text{sign}(x_{ik},y_{kj}) \neq \text{sign}(u_{il},v_{lj})$ (that is, α is sign-reversing outside its fixed point set);

(iii) α has no fixed points in I_{ij} for $i \neq j$, and a unique fixed point (with positive sign) in I_{ii} for each i;

α is an involution, that is, α^2 is the identity.

Then it is easy to see, by using α, that

$$\sum_{k=1}^{n} a_{ik} b_{kj} = \delta_{ij},$$

where δ_{ij} is the Kronecker delta.

Since $\mathbf{AB} = \mathbf{I}$ if and only if $\mathbf{BA} = \mathbf{I}$, it is natural to ask if $\mathbf{BA} = \mathbf{I}$ admits a combinatorial proof via a sign-reversing involution β defined on

$$J = \bigcup_{i,j=1}^{n} J_{ij},$$

where

$$J_{ij} = \bigcup_{k=1}^{n} B_{ik} \times A_{kj},$$

where β is constructed directly from α. We would also like the construction to be *natural*, in the sense that the map $\alpha \to \beta$ itself should be an involution.

Problem: Give such a construction of β from α.

Problem BCC12.9: Subgraphs of the *n*-cube. Proposed by Paul Erdős. Correspondent: Paul Erdős.

Let Q_n denote the n-cube, a graph with $n2^{n-1}$ edges.

(a) Show that a subgraph of Q_n with at least $(\frac{1}{2}+\varepsilon)n2^{n-1}$ edges contains a C_4.

(b) Show that a subgraph of Q_n with at least $\varepsilon n 2^{n-1}$ edges contains a C_6.

Of course, the assertion is to be shown for all sufficiently large n, for any given $\varepsilon > 0$. Each of these problems is worth $100.

Editor's Note: Part (b) has been disproved by Conder [35], who partitioned the edge-set of Q_n into three C_6-free subsets. For up-to-date results on both problems, see Chung and Graham [33], p. 43.

Problem BCC12.10: Sum-free sets containing a small even number. Proposed by Paul Erdős. Correspondent: Paul Erdős.

Let S be a sum-free subset of $\{1,\ldots,n\}$ (that is, for all $x,y,z \in S$, $x+y \neq z$). Clearly $|S| \leq \lceil \frac{1}{2}n \rceil$. Now suppose that S contains an even number less than $\frac{1}{2}n - f(n)$. Is it true that $|S| \leq \frac{1}{2}n - \varepsilon f(n)$?

The hypothesis is clearly necessary: the set of odd numbers, and the set of numbers greater than $\frac{1}{2}n$, are both sum-free. It is known that, if $2 \in S$, then $|S| \leq (\frac{2}{5} + o(1))n$.

Editor's Note: Deshouillers, Freiman, Sós and Temkin [38] have shown that a sum-free set S which contains an even number satisfies either $|S| \le \min(S)$ or $|S| \le \frac{2}{5}(n+2)$. More generally, they have given a description of all such sets satisfying $|S| > \frac{2}{5}n - c$ with $n > n_0(c)$ for fixed c.

Problem BCC12.11: Forbidding $x+y=4z$. Proposed by Paul Erdős. Correspondent: Paul Erdős.

Let S be a set of positive integers containing no solution to $x+y=4z$. Does S have lower density less than $\frac{1}{2}$?

Note that the set of integers congruent to 1 or 4 modulo 5 satisfies the constraint and has density $\frac{2}{5}$.

Problem BCC12.12: Sets with all subset-sums distinct. Proposed by Paul Erdős. Correspondent: Paul Erdős.

Let S be a subset of $\{1,\ldots,n\}$ with all sums of subsets distinct. Is it true that

$$|S| < \frac{\log n}{\log 2} + c?$$

It is known that $|S| < \frac{\log n}{\log 2} + \frac{\log\log n}{2\log 2}$, and that there is such a set with $n = 2^m$ and $|S| = m+2$. This problem is worth $500.

Problem BCC12.13: Some distance-regular graphs. Proposed by Tony Gardiner. Correspondent: Tony Gardiner.

Is there a distance-regular graph with diameter 3 having intersection array

$$\begin{pmatrix} * & 1 & c & m \\ 0 & 0 & a & 0 \\ m & m-1 & b & * \end{pmatrix}$$

with $b \ne 1$, $a \ne 0$?

The ith column of the intersection array gives the number of vertices adjacent to y and at distance $i-1$, i, $i+1$ respectively from x, where x and y are at distance i these numbers being (by definition) constant. The smallest such feasible graph has valency 126.

Problem BCC12.14: Determining a graph from its cycle space. Proposed by Oliver Pretzel. Correspondent: Oliver Pretzel.

Given a cycle basis of a graph G, one can view the cycles of G as elements of a free abelian group (or, if orientation is disregarded, of a vector space over $\mathbb{Z}/2$). Many functions on the cycles are linear on this algebraic structure, but the length function (the number of edges in the cycle) is not. Informally, the question is: "how closely does the length function determine G?"

More formally, suppose that two graphs G and H are given with isomorphic cycle spaces. Suppose further that G is known and that, for suitable cycle bases, the length functions on the cycle spaces of G and H are the same. What can be said about H?

For example, if G is a tree, then H is a forest. If G is a cycle with two chords and H is 2-connected, then H is a cycle of the same length with two chords, but the chords can be moved slightly. So H need not be isomorphic to G.

Conjecture: If G and H are 3-connected and have the same length function for cycles, then G and H are isomorphic

Problem BCC12.15: A double Youden rectangle. Proposed by Donald A. Preece. Correspondent: Donald A. Preece.

Construct a 5×11 double Youden rectangle.

For the definition, see Bailey [6]. Briefly: what is required is a 5×11 rectangle, each cell containing a symbol from $\{a_1, \ldots, a_{11}\}$ and a symbol from $\{b_1, \ldots, b_5\}$, such that

(i) the a's form a Latin rectangle, and the sets of a's appearing in the columns are the blocks of a symmetric $(11, 5, 2)$ BIBD;

(ii) each b appears once in each column and once paired with each a;

(iii) For $i \neq j$, the numbers of occurrences of b_i and b_j in any row differ by at most 1.

Of course, this definition is much less "symmetrical" than that in Bailey [6].

Editor's Note: This problem has been solved by the proposer [89].

Problem BCC12.16: Monotone directed paths in tournaments. Proposed by V. Linek, B. Sands, N. Sauer and R. E. Woodrow. Correspondent: Bill Sands.

Problem 1: If the edges of a tournament are coloured with three colours, is there a set of three vertices such that there is a monochromatic directed path from any other vertex to one of these three?

It is known that, if only two colours are used, then a single vertex will suffice. For three colours, three vertices are necessary but it is not known whether any finite number is sufficient. See [95].

Problem 2: Let P be a poset, and colour the edges of a tournament with the elements of P. Call a directed path $x_1, x_2, \ldots, x_k$ *monotone* if

$$\text{colour}(x_i x_{i+1}) \leq \text{colour}(x_{i+1} x_{i+2})$$

for all i. If P is the poset $\{\overset{\bullet}{\underset{\bullet}{|}}\ \ \bullet\}$, is there a set of three vertices of T such that there is a monotone directed path from any other vertex of T to one of these three?

Again it is known that two vertices will not always do, but no fixed number is known to suffice. Problem 1 is just Problem 2 with P a 3-element antichain. Linek and Sands [75] have found all posets P for which a *single* vertex suffices; they are exactly those containing neither $\{\bullet\ \ \bullet\ \ \bullet\}$ nor $\{\overset{\bullet}{\underset{\bullet}{|}}\ \ \bullet\}$.

Problem 3: Problem 2 can be generalised by replacing the poset P by a directed graph D, and replacing "monotone" paths by paths $x_1, \ldots, x_k$ such that

$$(\text{colour}(x_i x_{i+1}), \text{colour}(x_{i+1} x_{i+2})) \text{ is an edge of } D$$

for all i. For example, if $D = C_5$, the undirected 5-cycle (with each edge directed both ways), is there always a vertex in any tournament reachable from all others by a D-path?

Problem BCC12.17: Independence numbers in non-Hamiltonian graphs. Proposed by Douglas R. Woodall. Correspondent: Douglas R. Woodall.

Let G be a k-connected graph with vertex independence number $\alpha(G)$.

Conjecture 1: If G is not Hamiltonian, C is a longest circuit in G, and $k \geq 2$, then

$$\alpha(G - C) \leq \alpha(G) - k.$$

Conjecture 2: If G is not traceable, P is a longest path in G, and $k \geq 1$, then

$$\alpha(G - P) \leq \alpha(G) - k - 1.$$

Here, $G - C$ means the graph obtained by removing the *vertices* of C from G.

If true, both conjectures are best possible: let G be the union of a large number of complete graphs, all having exactly k vertices in common. Conjecture 2 is obvious if $k = 1$ (since the end vertices of P are not adjacent to each other or to anything outside P), and has been proved for $k = 2$, but is open for $k \geq 3$, as is Conjecture 1 in all cases.

Problem BCC12.18: Zara graphs. Proposed by François Zara. Correspondent: François Zara.

A *Zara graph* is a graph with the properties that, for some natural numbers r and t,

(i) each maximal clique is of cardinality r;

(ii) if C is a maximal clique and x a vertex not in C, then x is adjacent to exactly t vertices in C.

Problems:

1. Find new Zara graphs.

2. A Zara graph with $t = 2$ is a special kind of "extended generalised quadrangle" (see [29]). Can these graphs be classified?

For further background, see [14, 109, 110].

Editor's note: At the Second Franco-Chinese Conference on Counting and Coding, held at ENST, Paris, 6–8 September 1989, Tayuan Huang (Department of Applied Mathematics, National Chiao-Tung University, Hsin Chu 30050, Taiwan, ROC; thuang@twnctu01.bitnet) proposed a solution to Problem 1, involving a family of Zara graphs with $r = q^3$, $t = q^2$ for any prime power q, constructed from alternating bilinear forms in 4 variables over $GF(q)$. (Join two forms if their difference is singular.) Subsequently, it turned out that these are isomorphic to some of the known examples.

Of course, by definition, Problem 1 remains open!

Final note. Participants at the problem session will recall some discussion of a problem posed by Oliver Pretzel. Subsequently, a message reached me by an indirect route from Noga Alon, pointing out that the problem had been solved "decades ago" by Erdős and Selfridge. The proof is elegant, and participants may be interested in seeing it.

The problem was as follows. Let A and B be distinct finite sets of positive integers for which the multisets

$$A_2 = \{a + a' : a, a' \in A, a \neq a'\}, \quad B_2 = \{b + b' : b, b' \in B, b \neq b'\}$$

coincide. Is it true that $|A| = |B|$ is a power of 2?

Solution. Put $f(x) = \sum_{a \in A} x^a$ and $g(x) = \sum_{b \in B} x^b$. Then

$$\begin{aligned} f(x)^2 - f(x^2) &= 2 \sum_{r \in A_2} x^r, \\ g(x)^2 - g(x^2) &= 2 \sum_{s \in B_2} x^s. \end{aligned}$$

Hence $f(x)^2 - f(x^2) = g(x)^2 - g(x^2)$, that is, $(f-g)(x)(f+g)(x) = (f-g)(x^2)$. Suppose that $(f-g)(x) = (1-x)^k P(x)$ where $P(1) \neq 0$. Then

$$(f+g)(x) + \frac{(f-g)(x^2)}{(f-g)(x)} = (1+x)^k \frac{P(x^2)}{P(x)}.$$

Put $x = 1$ to get

$$2|A| = (f+g)(1) = 2^k \frac{P(1)}{P(1)} = 2^k,$$

so the answer is affirmative.

2 BCC13

BCC13 was held at the University of Surrey, Guildford, 8-12 July 1991. The contributed papers were published in *Discrete Mathematics* **125** (1994). The problems appear on pages 407–417 of the journal.

Problem BCC13.1 (DM178): Stable partitions of graphs. Proposed by Z. Füredi. Correspondent: Z. Füredi.

A partition of the vertex set V of a (simple) graph G into two non-empty parts V_1 and V_2 is called *stable* if, for $i = 1, 2$ and all vertices $x \in V_i$,

$$\deg_{G|V_i}(x) > \tfrac{1}{2} \deg_G(x).$$

A graph with a stable partition is called *stable*. The complete graph is not stable, but every 3-regular graph other than K_4 is stable.

What is the number of stable graphs? In particular, is the proportion of graphs on n vertices which are stable $o(1)$?

Problem BCC13.2 (DM179): Ramsey perfect graphs. Proposed by J. Nešetřil. Correspondent: J. Nešetřil.

Is it true that, for every perfect graph G, there exists a perfect graph H such that, for every partition $E(H) = E_1 \cup E_2$, there is an induced subgraph G' of H such that G' is isomorphic to G and $E(G') \subseteq E_i$ for some i ($i = 1$ or 2)?

We expect a negative answer to this question, despite the fact that the analogous question for partitions of vertices has an affirmative answer (using the Lovász Multiplication Lemma).

Problem BCC13.3 (DM180): Reconstructing spanning trees. Proposed by W. L. Kocay. Correspondent: C. Wakelin.

Is the number of spanning trees with exactly d automorphisms reconstructible from the deck of vertex-deleted subgraphs of a graph, for each d?

Problem BCC13.4 (DM181): 1 time 2-tough implies 2 times 1-tough?. Proposed by C. Hoede. Correspondent: C. Hoede.

Prove that the edge-set of a 2-tough graph can be partitioned into two sets E_1 and E_2 such that each of E_1 and E_2 induces a 1-tough spanning subgraph of G.

If this conjecture is true, then a 4-regular 2-tough graph would consist of two edge-disssjoint Hamiltonian cycles. One could accordingly extend Thomassen's conjecture to the assertion that any 2-tough graph contains two edge-disjoint Hamiltonian cycles.

[A graph is *t-tough* is, for any vertex cutset Sm $G-S$ has at most $|S|/t$ components. Chvátal conjectured that, for $t > \frac{3}{2}$, a t-tough graph is Hamiltonian. This was refuted by Thomassen, who conjectured instead that a 2-tough graph is Hamiltonian.]

Problem BCC13.5 (DM182): Path-tough graphs. Proposed by I. Schiermeyer. Correspondent: I. Schiermeyer.

Let G be a simple graph on n vertices, where $n \geq 3$, which is *path-tough* (that is, $G-v$ has a Hamiltonian path for any vertex v). Suppose that $d(u) = d(v) = d(w) \geq n-2$ for any three independent vertices u, v, w. Prove that either G is Hamiltonian, or G is isomorphic to the Petersen graph.

Problem BCC13.6 (DM183): Hamiltonian cycles and a bit more. Proposed by R. Häggkvist. Correspondent: R. Häggkvist.

Prove that any graph of order n amd minimum degree at least $\frac{1}{2}(n+1)$ contains a subgraph which is a Hamiltonian cycle together with a longest diagonal.

Problem BCC13.7 (DM184): The second longest cycle. Proposed by R. Häggkvist. Correspondent: R. Häggkvist.

Give lower bounds for the length of the second longest cycle in a Hamiltonian 3-regular graph. (The best bound currently is $n-4\sqrt{n}$.) More generally, the same problem with "minimum degree 3" in place of "3-regular".

Problem BCC13.8 (DM185): 4-chromatic covering graphs. Proposed by D. Youngs. Correspondent: D. Youngs.

What is the smallest 4-chromatic covering graph? (A graph is a *covering graph* if its edges can be directed in such a way that it becomes the Hasse diagram of a poset.) The answer lies between 12 and 14 inclusive.

Problem BCC13.9 (DM186): Special Hamiltonian paths. Proposed by W. T. Trotter and S. Felsner. Correspondent: W. T. Trotter.

Consider the diagram of the poset of all subsets of $\{1,2,\ldots,n\}$. As a graph, this is the n-cube. Does the graph have a Hamiltonian path $A_1, A_1, \ldots, A_{2^n}$ starting at $A_1 = \emptyset$ and having the following property?

> If, at step i, you visit a set A_i, then you must previously have visited all subsets of A_i with at most one exception. If there is an exception, you must visit it next (that is, it is A_{i+1}).

For example, with $n = 4$,

$$\emptyset, 1, 12, 2, 23, 3, 34, 4, 24, 124, 14, 134, 13, 123, 1234, 234$$

is such a path.

Problem BCC13.10 (DM187): Interval-regular graphs. Proposed by H. M. Mulder. Correspondent: H. M. Mulder.

Let G be a connected simple graph. The *interval* $I(u,v)$ between vertices u and v is the set of all vertices which lie on some shortest path form u to v. The graph is called *interval-regular* if, for any two vertices u and v, we have

$$||N(u) \cap I(u,v)| = d(u,v),$$

where $N(u)$ is the set of neighbours of u.

Examples of interval-regular graphs include hypercubes, and the 2-cube and 3-cube with added edges joining all vertices in $N(u)$ for some vertex u. The class of interval-regular graphs is closed under taking Cartesian products. See [85].

Conjecture: Let G be interval-regular. Then

$$x, y \in I(u,v) \Longrightarrow I(x,y) \subseteq I(u,v)$$

for any vertices u and v.

Problem BCC13.11 (DM188): Permutations of a cube. Proposed by M. K. Siu. Correspondent: M. K. Siu.

Let C be the n-cube graph, d the graph metric. Is every permutation of the vertices of c the composite of at most n permutations s_i, each satisfying

$$d(x, s_i(x)) \leq 1$$

for all $x \in C$?

Editor's Note: More generally, which graphs of diameter n have this property?

Note that it is not true that a permutation s he vertices of C which satisfies $d(x, s(x)) \leq k$ can be written as the product of at most k permutations s_i as above.

Problem BCC13.12 (DM189): Permutations of projective space. Proposed by A.Gyárfás. Correspondent: P. J. Cameron.

For which n and q does there exist a permutation π of the point set of $PG(n,q)$ with the property that, for any hyperplane H, there exists a hyperplane H' with $\pi(H) \cap H' = \emptyset$? It is known that the answer is "yes" for $n = 2$ (asymptotically, almost all permutations have this property), and "no" for $n > q$ (by a short argument due to A. Blokhuis).

Problem BCC13.13: *q*-polynomials. Proposed by A. Bonisoli. Correspondent: A. Bonisoli.

A polynomial $c(z) = \sum c_i z^i$ ovber $GF(q)$ is said to be a *q-polynomial* if $c_i \neq 0$ only if i is a power of q. (See [74].) Let $m = 2^d - 1$ be a Mersenne prime. Does there exist a 2-polynomial

$$c(z) = \sum_{i=0}^{d} x_i z^{2^i}$$

of degree 2^d such that $c(z)/z$ is irreducible over $GF(2)$? The answer is "yes" for the first four Mersenne primes, viz., 3, 7, 31, 127.

Problem BCC13.14 (DM190): Cyclic shifts of binary words. Proposed by P. J. Cameron. Correspondent: P. J. Cameron.

For odd n, let W_n be the space of binary words of length n and even weight. Let $f(n)$ be the maximum codimension of a subspace U of W_n such that the union of all cyclic shifts of U is equal to W_n. It is known that $f(n) \geq 2$ for $n > 3$, but little more is known. Does $f(n) \to \infty$ as $n \to \infty$? Or is $f(n) = 2$ for infinitely many n?

Problem BCC13.15 (DM191): A generalisation of arcs. Proposed by J. Bierbrauer. Correspondent: J. Bierbrauer.

Let $(V,\mathcal{B})$ be an affine plane of odd order q. Given a function $w : V \to \mathbb{Z}$ such that

$$\sum_{p\in L} w(p) \leq 2$$

for all $L \in \mathcal{B}$, set

$$\mathrm{mass}(w) = \sum_{p\in V} w(p).$$

Then $\mathrm{mass}(w) \leq q+2$ holds. The value $q+1$ can be attained (by the characteristic function of a conic). Prove that necessarily $\mathrm{mass}(w) \leq q+1$.

Problem BCC13.16 (DM192): Perfect Steiner triple systems. Proposed by C. J. Colbourn and A. Rosa. Correspondent: A. Rosa.

Let $(V,\mathcal{B})$ be a Steiner triple system of order v (a $\mathrm{STS}(v)$). For distinct points x,y contained in the block $\{x,y,z\}$, the *interlacing graph* G_{xy} is the 2-regular graph on the vertex set $V \setminus \{x,y,z\}$, in which a and b are adjacent whenever $\{a,b,x\}$ or $\{a,b,y\}$ is in $\mathcal{B}$. The STS is called *perfect* if G_{xy} is a Hamiltonian cycle for all $x,y \in V$.

Problem: Find more perfect STS. Are there infinitely many?

Only four are known, with orders 7, 9, 25 and 33. There is none of order 13 or 15. All known examples have point-transitive automorphism groups.

Problem BCC13.17 (DM193): A generalisation of affine designs. Proposed by P. J. Cameron and M. E. Kimberley. Correspondent: P. J. Cameron.

An *affine design* is a resolvable 2-design in which any two non-parallel blocks meet in a constant number y of points. What happens if we replace "non-parallel" by "non-disjoint" in this definition? In addition to affine designs, there are resolvable designs with $\lambda = 1$ (and $y = 1$), for example Kirkman systems. No others are known: the first unknown case is a resolvable 2-$(70,10,6)$ design with $y = 2$. See [24].

Problem BCC13.18 (DM194): Blocking-set-free configurations. Proposed by J. W. DiPaola and H. Gropp. Correspondent: H. Gropp.

A configuration n_3 has n points and n lines, with three points on each line and three lines through each point, so that two points lie on at most one line. A *blocking set* is a set of points meeting every line but containing none. For which $n \geq 7$ does there exist a connected n_3 configuration with no blocking sets?

It is known that such configurations exist for all but finitely many values of n. They do not exist for the falues 8–12 or 14. The values in doubt are 15–18, 20, 23, 24, 26, 29, 30, 32, 38, 44. See [40].

Editor's Note: See also BCC14.14 and BCC16.20.

Problem BCC13.19 (DM195): Permutations with few distances. Proposed by P. J. Cameron. Correspondent: P. J. Cameron.

For fixed s and large n, the best known upper and lower bounds for the maximum number of permutations of an n-set with s different distances are both roughly $(cn/s)^{2s}$ (with different values of c). (See [23].) Find the correct value.

Problem BCC13.20 (DM196): Partial transversals of Latin rectangles. Proposed by A. J. W. Hilton. Correspondent: A. J. W. Hilton.

Let R be an $n \times 2n$ Latin rectangle on $2n$ symbols. A partial transversal T of size s of R is a collection of s cells, no two in the same row or column, and no two containing the same symbol. Is it true that R can be expressed as the union of $2n$ partial transversals of size n?

An equivalent formulation: Call two $n \times 2n$ Latin rectangles R, S on the same set of symbols *orthogonal* if the pairs (r_{ij}, s_{ij}), for $i = 1, \ldots, n$ and $j = 1, \ldots, 2n$, are all distinct. Does every $n \times 2n$ Latin rectangle have an orthogonal mate?

Problem BCC13.21 (DM197): Semi-Latin squares. Proposed by R. A. Bailey. Correspondent: R. A. Bailey.

A *semi-Latin square* is an $n \times n$ array with k symbols (chosen from a set of size nk) in each cell, such that each symbol occurs once in each row or column. We impose the further property:

No two symbols occur together in a cell more than once.

Such a structure clearly exists if there are k mutually orthogonal Latin squares of order n. on disjoint sets of symbols.

Problem: Find constructions for values (n, k) for which a set of k m.o.l.s. of order n does not exist (or is unknown). Examples are known for $(n, k) = (6, 2)$ or $(6, 3)$. What about $(6, 4)$ or $(10, 3)$?

Editor's Note: See comments on BCC12.1 above.

Problem BCC13.22 (DM198): Tiling the square. Proposed by D. Youngs. Correspondent: D. Youngs.

(a) What is the least *odd* number of congruent non-rectangular tiles needed to tile a square?

(b) Is there such a tiling in which the tiles are not polyominoes?

The best tiling known to the proposer uses 25 copies of the polyomino with two rows containing 6 and 3 squares, aligned at one end.

Problem BCC13.23 (DM199): Covering the square. Proposed by F. Barnes. Correspondent: F. Barnes.

(a) Prove that, for any partition of the plane into sets (or regions) of diameter 1, the density must be at least $8/3\sqrt{3}$.

(b) A finite variation. What is the largest square which can be partitioned into n sets of diameter (at most) 1? The answer is known for $n \leq 5$. In general, we would expect a hexagonal honeycomb with some distortion at the edges.

An alternative formulation of (b) asks for the chromatic number of the graph whose vertices are the points of the unit square, two points adjacent if their distance exceeds d.

Problem BCC13.24 (DM200): Sum-free sets containing 2. Proposed by P. J. Cameron. Correspondent: N. J. Calkin.

What is the probability that, in a random sum-free set S of natural numbers, 2 is the only even number in S? (Is it zero or not?)

The probability measure is defined by the following rule. Consider the natural numbers in their usual order. If n is the sum of two numbers in S, then $n \notin S$; otherwise, decide on the toss of a fair coin. It is known that the probability that S contains no even number is non-zero, but the present problem seems a bit more delicate.

Editor's Note: This problem has been solved by the proposer and the presenter [21], who showed that the probability is strictly positive and gave a heuristic estimate for it.

Problem BCC13.25 (DM201): Partitions of intersecting families. Proposed by N. Alon, P. Seymour and Z. Füredi. Correspondent: Z. Füredi.

Let $\mathcal{F}$ be an intersecting family of k-subsets of the n-element set V, that is, $F \cap F' \neq \emptyset$ for all $F, F' \in \mathcal{F}$. Let $p(\mathcal{F})$ be the minimum p for which one can find p pairs (2-subsets) $P_!, \ldots, P_p$ of V such that every member of $\mathcal{F}$ contains some P_i. Now let $f(n,k)$ be the maximum of $p(\mathcal{F})$, over all such intersecting families. Is it true that $f(n,k) \leq n$ for all n?

If true, this would imply a strengthened form of the case $t+1$ of Larman's conjecture, since, setting

$$\mathcal{F}_i = \{F \in \mathcal{F} : p_i \subseteq F, P_j \not\subseteq F \text{ for } j < i\},$$

then $\{\mathcal{F}_i : i = 1, \ldots, n\}$ is a decomposition of $\mathcal{F}$ into 2-intersecting families.

(*Larman's conjecture* [72] asserts that, if $\mathcal{F}$ is a t-intersecting family of k-subsets of the n-set V, that is, $|F \cap F'| \geq t$ for all $F, F' \in \mathcal{F}$, then $\mathcal{F}$ can be decomposed into n subfamilies each of which is $(t+1)$-intersecting.

Editor's note: A similiar question can be asked for arbitrary families of sets (not all of the same size).

Problem BCC13.26 (DM202): Some families of sets. Proposed by N. J. Calkin. Correspondent: N. J. Calkin.

What can be said about families $\mathcal{F}$ of subsets of an n-set V such that

(a) $F_1, F_2 \in \mathcal{F} \implies F_1 \not\subset F_2$;

(b) $F_1, F_2 \in \mathcal{F} \implies F_1 \cap F_2 \neq \emptyset$;

(c) $(\forall x \in V)(\exists F_1, F_2 \in \mathcal{F})(F_1 \cap F_2 = \{x\})$?

Problem BCC13.27 (DM203): Some problems on perfect groups. Proposed by J. Dénes and P. Yff. Correspondent: P. Yff.

(a) Prove the Feit–Thompson Theorem by elementary means. (Remark: either of the following equivalents of the Feit–Thompson Theorem may be more convenient:

- A finite perfect group can be generated by a self-inverse conjugacy class of elements of odd order (Heineken, unpublished);
- If G has odd order n, then some element of G is not the product of n distinct factors (Dénes and Hermann [39]).)

(b) Show that a perfect group can be generated by an involution and an element of odd order which is conjugate to its inverse.

(c) Characterise finite groups in which every element is a commutator. In particular, show that every non-abelian finite simple group has this peroperty (*Ore's conjecture*).

(d) Characterise finite groups G in which every element of the derived group is the product of k commutators. (This condition can be expressed in terms of the notion of k-conjugacy (Yff [107]).

3 BCC14

BCC14 was held at the University of Keele, 5–9 July 1993. The contributed papers were published in *Discrete Mathematics* **138** (1995). The problems appear on pages 405–411 of the journal. The problems are © Elsevier Science B.V. 1995.

Problem BCC14.1 (DM215): Total colourings of hypergraphs. Proposed by P. Cowling. Correspondent: P. Cowling.

A *total colouring* of a hypergraph is a colouring of vertices and edges such that

(a) the restrictions to vertices and edges are strong colourings;

(b) an incident vertex and edge have different colours.

The *total chromatic number* is the least number of colours required for a total colouring. See [36].

Conjecture: If $H = (V, \mathcal{E})$ is a linear hypergraph (two vertices on at most one edge) with total chromatic number $\chi_T(H)$, then

$$\chi_T(H) \leq \max_{x \in V} \left| \bigcup_{\substack{E \in \mathcal{E} \\ x \in E}} E \right| + 1.$$

Problem BCC14.2 (DM216): Critical K_l-free graphs. Proposed by J. Schöhheim. Correspondent: J. Schöhheim.

It is known that k-chromatic critical graphs on n vertices have at least $\binom{k-1}{2} n$ edges. Gallai showed that a better lower bound holds for graphs containing no K_k. Can this bound be further improved for graphs containing no K_l, for fixed l with $3 \leq l \leq k$?

For example, with $k = 10$, $l = 9$, can the bound $4.5n$ be improved to $5n - 10$?

Problem BCC14.3 (DM217): Bandwidth of a graph. Proposed by D. B. West. Correspondent: D. B. West.

The *bandwidth* of an n-vertex graph G is

$$\min_f \max_{x \sim y} |f(x) - f(y)|,$$

where the minimum is over all bijections from the vertex set to $\{1, \ldots, n\}$.

What is the bandwidth of the "triangular lattice" graph whose vertices are all triples of non-negative integers with sum l, vertices (x,y,z) and (x',y',z') being adjacent whenever $|x-x'| + |y-y'| + |z-z'| = 2$? (A lower bound of $l/2$ is known, and an upper bound of $l+1$ is obtained by numbering the vertices in layers.)

Editor's Note: This problem was solved immediately after the conference. The solution [62] appears in the volume of contributed papers.

Problem BCC14.4 (DM218): How small is Tutte's wheel?. Proposed by A. Shastri. Correspondent: A. Shastri.

W. T. Tutte proved that any 3-connected graph can be obtained from a wheel by repeatedly adding an edge or splitting the central vertex (keeping the minimum degree at least 3).

Conjecture. Any 3-connected cubic graph on n vertices may be obtained by this procedure from a wheel on k vertices, where $k \geq cn$ (for some absolute constant c).

Problem BCC14.5 (DM219): Characteristic polynomials of graphs. Proposed by R. Häggkvist. Correspondent: R. Häggkvist.

How many distinct characteristic polynomials of (adjacency matrices of) n-vertex graphs are there?

The proposer conjectures that a typical n-vertex graph has n^2 cospectral mates, so that the answer to the problem is $O(2^{n(n-1)/2}/n^2 n!)$.

Problem BCC14.6 (DM220): Cliques and cocliques in Cayley graphs. Proposed by N. Alon. Correspondent: N. Alon.

Conjecture. There is a constant c such that, for every finite group G of order $n > 1$, there is a symmetric (i.e., inverse-closed) generating set S for G such that the Cayley graph $\Gamma(G,S)$ has neither a clique nor an independent set of size $c\log n$.

This is not known for any infinite sequence of finite groups; but it is true with $\log^2 n$ replacing $\log n$.

Problem BCC14.7 (DM221): Local structure in 2-transitive graphs. Proposed by A. A. Ivanov. Correspondent: A. A. Ivanov.

Problem: Determine the vertex and edge stabilizers in all locally finite 2-transitive graphs in which $G_1(x) = 1$.

(A graph is 2-transitive if it admits a group G acting transitively on 2-arcs. The condition $G_1(x) = 1$ means that a vertex stabilizer acts faithfully (and 2-transitively) on its neighbours. The answer to this problem would be a list of pairs (H,t), where H is a finite 2-transitive group (the vertex-stabiliser $G(x)$) and t an outer automorphism of order 2 of the stabilizer H_y (so that $H_y\langle t\rangle = G(e)$, where $e = \{x,y\}$), along with the trivial possibility that $G(e) = H_y \times 2$.)

Problem BCC14.8 (DM222): The rows of a Latin square. Proposed by P. J. Cameron and J. C. M. Janssen. Correspondent: P. J. Cameron.

(a) It is known that, for almost all Latin squares of order n (that is, a proportion tending to 1 as $n \to \infty$), the rows of the square (regarded as permutations) generate S_n or A_n. Is this statement still true if the squares are normalized so that the first row is the identity permutation?

(b) Is it true that the distribution of the number of rows of a random Latin square which are odd permutations is "approximately" binomial $B(n, \frac{1}{2})$?

(c) Let $M(n)$ and $m(n)$ denote the maximum and minimum numbers of extensions of a $2 \times n$ Latin rectangle to an $n \times n$ Latin square. Find a good upper bound for $M(n)/m(n)$.

(d) How do you choose a random Latin square of order n?

Editor's Note: In connection with (b), Häggkvist and Janssen [56] have shown that the proportion of Latin squares in which all rows are even permutations is exponentially small. This was the form asked at the Conference; the strengthened version was suggested by Jeannette Janssen.

A Markov chain method for choosing a random Latin square was given by Jacobson and Matthews [63].

Problem BCC14.9 (DM223): A bijective proof of the Dyson conjecture. Proposed by R. Lewis. Correspondent: R. Lewis.

Let $R(r,m,n)$ denote the set of partitions of n whose rank is congruent to r modulo m, where the *rank* of a partition is the largest part minus the number of parts. Freeman Dyson conjectured, and Atkin and Swinnerton-Dyer [5] proved, that

$$|R(0,5,5n+4)| = |R(1,5,5n+4)| = \ldots = |R(4,5,5n+4)|.$$

The problem is to find a bijective proof.

Problem BCC14.10 (DM224): Even and odd permutations. Proposed by P. J. Cameron. Correspondent: P. J. Cameron.

For even n, the number of permutations of $\{1,\ldots,n\}$ with all cycles of even length is equal to the number of permutations with all cycles of odd length. Find a bijective proof of this fact.

Editor's Note: After the conference, this problem was solved independently by Richard Lewis and Simon Norton. Their joint paper [73] appears in the volume of contributed papers.

Problem BCC14.11 (DM225): How many sum-free sets?. Proposed by P. J. Cameron and P. Erdős. Correspondent: P. J. Cameron.

Let $s(n)$ be the number of sum-free subsets of $\{1,\ldots,n\}$ (that is, containing no solution to $x+y=z$). Show that there exist constants c_o and c_e such that $s(n)/2^{n/2} \to c_o$ or c_e as $n \to \infty$ through odd or even values respectively.

It is known only that $s(n) = 2^{(\frac{1}{2}+o(1))n}$: see [3, 20].

Editor's Note: For the motivation, and conjectured values of the constants c_e and c_o, see [27].

Problem BCC14.12 (DM226): Non-crossing queens. Proposed by G. B. Khosrovshahi. Correspondent: G. B. Khosrovshahi.

What is the maximum number of non-crossing n-queens? It is known that the maximum is n if n is prime.

Problem BCC14.13 (DM227): Block-transitive designs. Proposed by P. J. Cameron and C. E. Praeger. Correspondent: P. J. Cameron.

A t-(v,k,λ) design has v points and a collection of blocks of size k, any t points lying in exactly λ blocks. Terms such as "block-transitive" apply to the action of the automorphism group.

(a) Show that there is no block-transitive 6-design.

(b) Show that a block-transitive, point-imprimitive 3-design satisfies $v \leq \binom{k}{2} + 1$.

(c) Is there a block-transitive 2-$(\infty, 4, 1)$ design which is not point-transitive?

Editor's Note: For background to (a) and (b) see [30, 31]. The design asked for in (c) has been constructed by David Evans [46]. Part (b) has been answered in the affirmative by Avinoam Mann and Ngo Dac Tuan (to appear).

Problem BCC14.14 (DM228): Blocking-set-free configurations. Proposed by J. W. DiPaola and H. Gropp. Correspondent: H. Gropp.

A configuration n_3 has n points and n lines, with three points on each line and three lines through each point, such that two points lie on at most one line. A *blocking set* is a set of points meeting every line but containing none. For which $n \geq 7$ does there exist a connected n_3 configuration with no blocking sets?

It is known that such configurations exist for all but finitely many values of n. They do not exist for the values 8–12 or 14. The values in doubt are 15–18, 20, 23, 24, 26. (The value 15 may now be settled). See J. W. DiPaola and H. Gropp [40].

Editor's Note: This problem an updated form of BCC13.18. See also BCC16.20.

Problem BCC14.15 (DM229): Arranging rows and columns. Proposed by D. B. West. Correspondent: D. B. West.

A matrix of zeros and ones is said to be "zero-partitionable" if its rows and columns can be permuted independently so that the zeros of the resulting matrix can be labeled R or C such that

- every position to the right of an R is a 0 labeled R, and
- every position below a C is a 0 labeled C.

What is the complexity of recognizing zero-partitionable matrices?

This is equivalent to recognition of interval digraphs. If a 0 is allowed to receive both R and C, this becomes recognition of digraphs with Ferrers dimension 2, which runs in polynomial time.

4 BCC15

BCC15 was held at the University of Stirling, 3-7 July 1995. The contributed papers were published in *Discrete Mathematics* **167/168** (1997). The problems appear on pages 605–615 of the journal. The problems are © Elsevier Science B.V. 1997.

Problem BCC15.1 (DM271): A generalization of Hadwiger's conjecture. Proposed by Ding, Oporowski, Sanders, and Vertigan. Correspondent: D. P. Sanders.

A *vertex partition* of G is a set $\{A_1,\ldots,A_k\}$ of induced subgraphs such that $V(G)$ is the disjoint union $V(A_1)\cup\ldots\cup V(A_k)$.

Conjecture: Every graph with no K_n minor has a vertex partition into $n-m+1$ graphs with no K_m minor.

For $m=2$, this is Hadwiger's conjecture. It is known to be true for $n\leq 5$ (Wagner [103]; Ding, Oporowski, Sanders, Vertigan); for $n=6, m=2$ (Robertson, Seymour, Thomas [92]), and for $6\leq n\leq 8$, $m=3$ (Jørgensen [64]).

Problem BCC15.2 (DM272): Uniquely total colourable graphs. Proposed by M. Behzad and E. S. Mahmoodian. Correspondent: E. S. Mahmoodian.

A *total colouring* of a graph is a colouring of the vertices and edges in such a way that no two adjacent or incident elements have the same colour.

Problem: Show that, apart from empty graphs, paths, and cycles C_{3k}, there is no graph which has a unique total colouring (in the minimum number of colours).

A prize of 500000 Iranian rials is offered for this problem. See [78].

Problem BCC15.3 (DM273): 1-track-less orientations. Proposed by Jörg Zuther. Correspondent: Jörg Zuther.

A 1-*track* is a one-way infinite directed path (which may be directed either in or out).

Problem: Characterize those graphs which admit a 1-track-less orientation.

Note that every locally finite graph, and every m-partite graph (for finite m) has a 1-track-less orientation, but the countable complete graph does not.

Problem BCC15.4 (DM274): Continuous maps between graphs. Proposed by Anthony Hilton. Correspondent: Anthony Hilton.

A map is k-*to*-1 if the inverse of every point in the codomain has cardinality k. See [61].

Problem: Determine the triples (k,m,n) for which there is a k-to-1 continuous map from $K_{m,m}$ to $K_{n,n}$, where these graphs are regarded as 1-dimensional simplicial complexes in the usual way (with edges homeomorphic to $[0,1]$).

Problem BCC15.5 (DM275): A generalization of Tarsi's problem. Proposed by R. Klein and J. Schönheim. Correspondent: J. Schönheim.

A graph is *m-degenerate* if every subgraph has a vertex of valency at most m.

Problem: Prove or disprove that a graph which is the edge-disjoint union of subgraphs $G_1,\ldots,G_s$, where G_i is m_i-degenerate, can be coloured with

$$\sum_{i=1}^{s} m_i + \left\lfloor \tfrac{1}{2}\left(1+\sqrt{1+8\sum_{1\le i<j\le s} m_i m_j}\right)\right\rfloor$$

colours.

For $s=2$, $m_1=1$, $m_2=2$, this is M. Tarsi's problem [68].

Problem BCC15.6 (DM276): Common vertices on longest paths. Proposed by T. Gallai. Correspondent: Sandi Klavžar.

Let G be a finite connected graph. Do any three longest paths in G have a common vertex? It is trivially true that every two longest paths have a common vertex; but there are graphs in which no vertex lies on all the longest paths. (The problem is due to Gallai [49]; see also [108, 67]

Problem BCC15.7 (DM277): Edge-colourings of complete graphs. Proposed by Peter Johnson. Correspondent: Peter Johnson.

Suppose that the edges of the complete graph K_n ($n>1$) are coloured with four colours R, G, B, Y such that each colour-class gives a connected subgraph on n vertices. It is easy to see from Satz 1.2(3) of Gallai [48] that at least three of the four triangles with edge colourings RGB, RGY, RBY, GBY occur.

Questions:

- (a) Do all four occur?
- (b) If not, how small can n be?
- (c) What happens with more than four colours?

Problem BCC15.8 (DM278): On the probability of connectedness. Proposed by P. J. Cameron. Correspondent: P. J. Cameron.

Which graphs G have the property that, in the class $\mathcal{X}(G)$ of graphs having no induced subgraph isomorphic to G, the limiting probability of connectedness is strictly between zero and one (in either the unlabelled or the labelled case)? (The smallest G with this property is the path of length 3; the probability of connectedness in $\mathcal{X}(G)$ is $\frac{1}{2}$ if the number of vertices is greater than one.)

Editor's Note: See [12] for more information.

Problem BCC15.9 (DM279): Characteristic and chromatic polynomials. Proposed by Roland Häggkvist. Correspondent: Roland Häggkvist.

The *characteristic polynomial* of a graph G is the polynomial $\det(xI - A(G))$, where $A(G)$ is the adjacency matrix of G. Its roots are the *eigenvalues* of G.

Question: Are there more characteristic polynomials than chromatic polynomials of graphs on n vertices?

Editor's Note: See also BCC14.5.

Problem BCC15.10 (DM280): Graphs with three eigenvalues. Proposed by Willem Haemers. Correspondent: Willem Haemers.

Let G be a connected graph with just three distinct eigenvalues. Such a graph, if regular, must be strongly regular; and any strongly regular graph has this property. Non-regular examples include the complete bipartite graphs, and one further example on 36 vertices constructed by M. Muzychuk.

Questions:

(a) Is it true that G has at most two distinct valencies?

(b) Is G switching-equivalent to a null or strongly regular graph?

(c) Find more examples.

(The operation of *switching* a graph with respect to a set X of vertices replaces each edge from X to its complement by a non-edge and each such non-edge by an edge, leaving edges within or outside X unaltered.)

Editor's Note: M. Klin and M. Muzychuk [86] have pointed out that a family of examples were found in 1981 by Bridges and Mena [17], and have also constructed some 'sporadic' examples and re-formulated and analysed the question.

Problem BCC15.11 (DM281): Spectra of K_{s+1}-free graphs. Proposed by Stephan Brandt. Correspondent: Stephan Brandt.

Let λ_1 and λ_n be the greatest and smallest eigenvalues of a graph on n vertices.

Conjecture: $(\lambda_1+\lambda_n)/n \le 4/25$ for any regular triangle-free graph on n vertices.

This conjecture would be true if any of the following two old Erdős conjectures holds (see e.g. [44]): Let G be a triangle-free graph on n vertices. Then (a) G contains a set of $\lfloor n/2 \rfloor$ vertices which span at most $n^2/50$ edges, and (b) G can be made bipartite by the omission of at most $n^2/25$ edges.

Problem: Let $\xi(s)$ be the supremum of $(\lambda_1+\lambda_n)/n$ over the class of regular K_{s+1}-free graphs on n vertices. Determine or estimate $\xi(s)$.

The author [16] can show that

$$\begin{aligned} 0.14 &\le \xi(2) \le 3-2\sqrt{2} = 0.1715\ldots \\ (s-2)/s &\le \xi(s) \le (s-2)/(s-1) \qquad \text{for } s \ge 3. \end{aligned}$$

Problem BCC15.12 (DM282): Semiregular automorphism groups. Proposed by Dragan Marušič, Mikhail Klin. Correspondent: Mikhail Klin.

A permutation group is *semiregular* if no non-identity group element fixes a point. It is *regular* if it is transitive and semiregular. A graph is a Cayley graph if and only if its automorphism group contains a regular subgroup. It is known that there are vertex-transitive graphs which are not Cayley graphs (the smallest such being the Petersen graph.)

Question: Is there a vertex-transitive graph whose automorphism group contains no non-identity semiregular subgroup?

More generally, is there a 2-closed transitive permutation group containing no non-identity semiregular subgroup? (A permutation group is 2-*closed* if it is the automorphism group, preserving the colours, of some edge-coloured directed graph.)

Editor's Note: Not every transitive permutation group contains a non-identity semiregular subgroup: the smallest counterexample has degree 12 (see [86]). Recently, Giudici [50] has determined all the quasiprimitive permutation groups which contain no non-identity semiregular subgroup; none of them is 2-closed.

Problem BCC15.13 (DM283): A distance-regular graph. Proposed by Leonard Soicher. Correspondent: Leonard Soicher.

Let C_{22} be the code obtained by puncturing the non-extended binary Golay code C_{23} in one coordinate. Then C_{22} is a $[22,12,6]$ code with automorphism group $M_{22}:2$. Let M be the set of words of minimum non-zero weight in C_{22}, so that $|M| = 77$.

Let V be the set of pairs $\{v_1, v_2\}$ of words of C_{22} which satisfy $\mathrm{wt}(v_1) = \mathrm{wt}(v_2)$ and $v_1 + v_2 = \mathbf{1}$, where $\mathbf{1}$ is the all-1 word. Then $|V| = 672$. For $v = \{v_1, v_2\} \in V$, define

$$M(v) = \{m \in M \,|\, \mathrm{wt}(v_1 + m) = \mathrm{wt}(v_2 + m)\}.$$

Then $|M(v)| = 55$ for all $v \in V$.

Define a graph Γ with vertex set V, in which $v \sim w$ if and only if $|M(v) \cap M(w)| = 43$. Then Γ is a distance-regular, but not distance-transitive graph. Moreover, the distance function in Γ is given by

$$d_\Gamma(v,w) = \tfrac{1}{4}(47 - |M(v) \cap M(w)|)$$

for $v, w \in V$, $v \neq w$.

These facts have been proved using the package **GRAPE** (see [98]).

Problem:

(a) Prove this by hand, to help understand Γ.

(b) Can a similar construction be applied to other codes with even length and minimum weight, to construct other distance-regular graphs?

Problem BCC15.14 (DM284): Pasch configurations in 3-hypergraphs. Proposed by G. B. Khosrovshahi. Correspondent: G. B. Khosrovshahi.

Let $X = \{1, 2, \ldots, v\}$. Denote the set of all 3-subsets of X by $P_3(X)$. Show that for $v \geq 6$, any $(\binom{v}{2} + 1)$-subset of $P_3(X)$ must contain a *Pasch configuration*, that is, $\{abc, axy, bxz, cyz\}$ for some $a, b, c, x, y, z \in X$.

Editor's Note: A. Blokhuis (personal communication) has constructed hypergraphs with more than $\binom{v}{2} + 1$ edges with no Pasch configuration. The question should be modified to read: how many edges can a hypergraph with no Pasch configuration have?

Problem BCC15.15 (DM285): Critical sets in Latin squares. Proposed by Ebad Mahmoodian. Correspondent: Ebad Mahmoodian.

A *critical set* in an $n \times n$ array with entries from the set $\{1, \ldots, n\}$ is a set S of the positions of the array with the property that the entries in the positions of S have a unique extension to a Latin square of order n.

Problem: Show that any critical set in a Latin square of order n has cardinality at least $\lfloor n^2/4 \rfloor$.

Problem BCC15.16 (DM286): Loops with conditions on area. Proposed by Alain Valette. Correspondent: Alain Valette.

A *loop* γ is a closed trajectory in the square lattice. Its *algebraic area* is $A(\gamma) = \oint_\gamma x\mathrm{d}y$. Let $N(2k;\mathcal{P})$ be the number of loops based at 0, of length $2k$, which satisfy property $\mathcal{P}$. So, for example, $N(2k;\emptyset) = \binom{2k}{k}^2$.

Problem: Find either formulae or asymptotics for $N(2k;A=l)$ and $N(2k;A \equiv l \pmod q)$, for given l,q.

For example, $\lim_{k\to\infty} N(2k;A=0)^{1/2k} = 4$. Also, if $n(2k,l,q)$ denotes $N(2k;A \equiv l \pmod q)$, then it is known that

$$\lim_{k\to\infty}(n(2k;0,2)-n(2k;1,2))^{1/2k} = 2\sqrt{2},$$
$$\lim_{k\to\infty}(n(2k;0,3)-n(2k;1,3))^{1/2k} = 1+\sqrt{3},$$
$$\lim_{k\to\infty}(n(2k;0,4)-n(2k;2,4))^{1/2k} = 2\sqrt{2}.$$

This problem (secretly) deals with the walk generating function of the discrete Heisenberg group in its 2-generator presentation.

Problem BCC15.17 (DM287): Proof of an identity. Proposed by Richard Lewis. Correspondent: Richard Lewis.

For complex numbers $z \neq 0$, $|w| < 1$, set $[z;w] = \prod_{n=1}^{\infty}(1-zw^{n-1})(1-z^{-1}w^n)$. It can be shown, using Cauchy's theorem, that for any non-zero complex numbers $a_1,\ldots,a_n,b_1,\ldots,b_n$ with $a_1\ldots a_n = b_1\ldots b_n$, and any q with $|q|<1$,

$$\sum_{r=1}^{n} \frac{[a_1b_r^{-1};q][a_2b_r^{-1};q]\ldots[a_nb_r^{-1};q]}{[b_1b_r^{-1};q][b_2b_r^{-1};q]\ldots\hat{}\ldots[b_nb_r^{-1};q]} = 0,$$

where the ˆ means to omit the term $[b_rb_r^{-1};q]$.

Problem: Find a combinatorial (bijective) proof of this inequality.

Problem BCC15.18 (DM288): Counting classes of graphs. Proposed by Peter Cameron. Correspondent: Peter Cameron.

Find good asymptotic estimates for the numbers of

(a) line graphs,

(b) line graphs of bipartite graphs,

(c) comparability graphs of 2-dimensional posets

on n vertices? (The last class of graphs are defined as follows: Take a permutation π of $\{1,\ldots,n\}$, and join i to j whenever $(i-j)(i\pi-j\pi)>0$.)

Problem BCC15.19 (DM289): Antichains in products of chains. Proposed by Jonathan D. Farley. Correspondent: Jonathan D. Farley.

Let $\theta(P)$ be the set of antichains of the poset P, and let $\underline{n}$ be the n-element chain. Dedekind's problem [69] asks for the value of $|\theta(\underline{2}^n)|$. It is easy to show that $|\theta(\underline{n})| = n+1$ and $|\theta(\underline{m}\times\underline{n})| = \binom{m+n}{m}$. MacMahon, Stanley [100, 101], Berman and Köhler [13] showed that

$$|\theta(\underline{k}\times\underline{m}\times\underline{n})| = \prod_{j=0}^{k-1}\binom{m+n+j}{m}\Big/\binom{m+j}{m}.$$

(Despite appearances, this function is symmetric!)

Problem: What is $|\theta(\underline{j}\times\underline{k}\times\underline{m}\times\underline{n})|$?

Problem BCC15.20 (DM290): Cycles of a permutation. Proposed by Peter Cameron. Correspondent: Peter Cameron.

As an example of a "typical" automorphism of the space of periodic integrable functions (acting on Fourier coefficients), W. Rudin [94] considered the permutation of the integers defined by

$$3n\mapsto 2n,\qquad 3n+1\mapsto 4n+1,\qquad 3n-1\mapsto 4n-1.$$

Problem: Describe the cycles of this permutation. In particular, does it have only finitely many finite cycles?

Editor's Note: This problem is older: it is the "original Collatz problem" from the 1930s (before the famous $3x+1$ problem), though never published by him. A paper by Jeff Lagarias [71] gives details.

Problem BCC15.21 (DM291): Combinatorics and control theory. Proposed by Holger Schellwat. Correspondent: Holger Schellwat.

In place of the Laplace transform, which is used to model continuous time control systems, in discrete control the $\mathcal{Z}$-transform is the basic tool. For a function $f : \mathbb{Z} \to \mathbb{C}$, its $\mathcal{Z}$–transform is defined by $\mathcal{Z}(f)(z) = \sum_{k=0}^{\infty} f(k)z^{-k}$. One problem, for example, is the stability of an inert linear controller and is determined by the loci of poles of a fraction of polynomials in $\mathbb{R}[x]$, constituting the $\mathcal{Z}$-transform of the transfer function [4]. On the other hand, the method of generating functions [104] is used widely in combinatorics to solve enumeration problems. If $(a_i : i \in \mathbb{N})$ is a sequence of numbers, for instance counting the number of distinct combinatorial objects of a certain kind, its associated ordinary generating function is the formal power series $\sum_{n=0}^{\infty} a_n z^n$. But up to the sign of the exponent, this is the defining sum for the $\mathcal{Z}$-transform of the sequence, viewed as a function. Thus it seems natural to explore the implications of this correspondence. Is it even possible to use it to translate problems in control theory into problems in combinatorics and/or vice versa? Could representation theory help to establish such a correspondence?

5 BCC16

BCC16 was held at Queen Mary and Westfield College, London, 7-11 July 1997. The contributed papers were published in *Discrete Mathematics* **197/198** (1999). The problems appear on pages 799–812 of the journal.

Problem BCC16.1 (DM315): Hamiltonian planar cubic graphs. Proposed by S. Jendrol' and Z. Skupień. Correspondent: S. Jendrol'.

Let G be a cubic bipartite 3-connected planar graph whose edge set $E(G)$ can be partitioned into three subsets $E(G) = E_1 \cup E_2 \cup E_3$ such that $\langle E_1 \rangle$ and $\langle E_2 \rangle$ are trees and $\langle E_3 \rangle$ is a cycle. Prove that G is hamiltonian.

Remark: This is true if E_2 is empty (Halin) or if E_2 consists of a single edge (Skupień).

Problem BCC16.2: Quasi Hamiltonian-Connected Graphs. Proposed by M. Alabdullatif. Correspondent: M. Alabdullatif.

Consider a Hamiltonian graph with the following property: for each pair $\{u, v\}$ of non-adjacent vertices, there exists a hamiltonian path joining u and v. Call such graph *quasi hamiltonian-connected* (*QHc*).

Unlike a hamiltonian-connected graph, a QHc graph may have connectivity 2. It has been shown that a k-regular QHc graph is necessarily 3-connected for $k = 3$ or 4 (see [2]).

Problem: Let G be a k-regular QHc graph ($k = 3$ or 4). Is G necessarily hamiltonian-connected?

The proposer checked the case when G is of order less than or equal to 10, and it turned out that G is hamiltonian-connected.

Remark: Counterexamples to this problem, both for $k = 3$ and for $k = 4$, have been found by Gunnar Brinkmann. The smallest counterexample has 16 vertices. (This information is from Stefan Brandt.)

Editor's Note: In the preliminary version, the proposer asked whether a 3-connected k-regular QHc graph is necessarily hamiltonian-connected? A negative answer was given by Stefan Brandt at the conference. He constructed, for each k, a k-connected, $(2k-1)$-regular graph which is QHc but not hamiltonian-connected.

Problem BCC16.3 (DM316): A condition for pancyclicity. Proposed by Uwe Schelten and Ingo Schiermeyer. Correspondent: Ingo Schiermeyer.

The *k-closure* $C_k(G)$ of a graph G was defined by Bondy and Chvátal (1976): recursively join all pairs of non-adjacent vertices whose degrees have sum at least k. They showed that, for an n-vertex graph G,

If $C_n(G) = K_n$ then G is Hamiltonian.

Faudree, Flandrin, Favaron and Li (1992) showed that

If $C_{n+1}(G) = K_n$ then G is pancyclic.

The examples $G = K_{n/2,n/2}$ (and many others) show that it is not true that if $C_n(G) = K_n$ then G is pancyclic.

Conjecture: If $C_n(G) = K_n$ and n is odd then G is pancyclic.

Problem BCC16.4 (DM317): Cutsets in bridged graphs. Proposed by Geña Hahn. Correspondent: Geña Hahn.

A graph G is *bridged* if every cycle of length of at least 4 has a bridge, that is, if every cycle C of length 4 contains two vertices u and v such that $d_G(u,v) < d_C(u,v)$ (distance).

Conjecture: For any pair of vertices u,v in a minimum cutset in a bridged graph G, $d_G(u,v) \leq 2$.

Problem BCC16.5 (DM318): 4-cycles in regular spanning subgraphs. Proposed by Ian Wanless. Correspondent: Ian Wanless.

Let n and k be integers satisfying $n \geq 2k$. Let G be a k-regular spanning subgraph of $K_{n,n}$ and let $s(G)$ be the number of 4-cycles in G. Suppose that G is chosen to maximise $s(G)$.

Conjecture: G contains $K_{k,k}$ as a component.

Notes: The conjecture is easily proved if

(1) k divides n, or

(2) n is sufficiently large.

If it is true *in toto*, then G is determined up to isomorphism. This follows from the fact that if G maximises $s(G)$ then $s(\overline{G})$ is also maximised, where $\overline{G}$ denotes the bipartite complement of G.

A weaker result along the same lines would be to show that G is necessarily disconnected.

Problem BCC16.6 (DM319): Alternating cycles in 2-arc-coloured tournaments. Proposed by G. Gutin, B. Sudakov and A. Yeo. Correspondent: Gregory Gutin.

A cycle C in a 2-arc-coloured digraph is alternating if any two consecutive arcs in C have different colours.

Problem: Does there exist a polynomial algorithm to check whether a 2-arc-coloured tournament has an alternating cycle?

The same problem for 2-arc-coloured digraphs has been proved to be NP-complete.

Problem BCC16.7 (DM320): Monochromatically absorbing sets in tournaments. Proposed by Geña Hahn. Correspondent: Geña Hahn.

Call a set S of vertices in an edge-coloured tournament *monochromatically absorbing* if, from any $u \in V(D) \setminus S$ there is a monochromatic directed path from u to S.

It follows from a theorem of Sands, Sauer and Woodrow [95] that any tournament not containing a ray (directed infinite path) whose arcs are coloured in two colours contains a vertex v such that $\{v\}$ is monochromatically absorbing.

Problems:

(i) Is there a function $f : \mathbb{N} \to \mathbb{N}$ such that in any tournament without a ray, edge-coloured in k colours, there is a monochromatically absorbing set of size at most $f(k)$? (It is known that if such a function exists, then $f(3) \geq 3$.)

(ii) Shen has shown that in a 3-arc-coloured finite tournament without 3-coloured triples (that is, such that between any three points two of the three arcs have the same colour) always contains a monochromatically absorbing vertex; this can be generalized to k colours. Is this true if the 3-coloured tournament has no 3-coloured directed cycles (as Shen [97] conjectures)?

(iii) If an infinite tournament without rays is arc-coloured with k colours, is there always a *finite* monochromatically absorbing set?

Editor's Note: See also BCC12.16.

Problem BCC16.8 (DM321): The ultimate independence ratio of a wheel. Proposed by Geña Hahn. Correspondent: Geña Hahn.

Define the *independence ratio* of a graph G by $i(G) = \alpha(G)/|G|$, and the *ultimate independence ratio* by $I(G) = \lim_{k\to\infty} i(G^k)$, with $G^1 = G$ and $G^k = G \Box G^{k-1}$.

Conjecture: $I(W_5) = I(W_{2n+1}) = \frac{1}{4}$ (where W_{2n+1} is the odd wheel).

Problem BCC16.9 (DM322): Colorings with minimum sum. Proposed by H. Hajiabolhassan, M.L. Mehrabadi, and R. Tusserkani. Correspondent: E.S. Mahmoodian.

Let G be a graph. A minimal coloring of G is a coloring which has the smallest possible sum among all proper colorings of G, using natural numbers. The *vertex–strength* of G, denoted by $s(G)$, is the minimum number of colors which is necessary to obtain a minimal coloring. Prove or disprove:

$$s(G) \le \left\lceil \frac{\chi(G) + \Delta(G)}{2} \right\rceil.$$

We have already proved that:

$$s(G) \le \lceil \frac{col(G) + \Delta(G)}{2} \rceil.$$

where $col(G)$ is the smallest number d such that for some linear ordering $<$ of the vertex set, the back degree $|\{v : v < u,\ vu \in E(G)\}|$ of every vertex u is strictly less than d. Therefore for all graphs G with $\chi(G) = col(G)$ e.g. all trees, the conjecture is verified. Also, we have verified this conjecture for line graphs.

Problem BCC16.10 (DM323): Colouring Kempe chains. Proposed by F. C. Holroyd, W. S. Leng. Correspondent: W. S. Leng.

Let G be a plane graph all of whose faces except the infinite face are triangles, and whose vertex set can be expressed as the disjoint union $V_0 \cup \ldots \cup V_n$, where

(i) V_0 is a single vertex and, for each $i = 1, \ldots, n$, V_i induces a cycle;

(ii) each vertex in V_i is adjacent only to vertices in V_{i-1}, V_{i+1} and two other vertices of V_i.

Can it be shown, without assuming the four-colour theorem, that there is a proper 4-colouring of G such that each V_i receives at most three colours?

Problem BCC16.11 (DM324): Linkages and flows. Proposed by Bruce Reed. Correspondent: Bruce Reed.

Find a short proof of the following theorem of Robertson and Seymour:

> *There is a function f such that, if the vertex set of a graph can be partitioned into a unique k-linkage (that is, k disjoint rooted paths as on page 101 of the proposer's paper [91] in the Proceedings), then it can be partitioned into a unique $f(k)$-flow.*

Problem BCC16.12 (DM325): Second neighbourhoods in digraphs. Proposed by Paul Seymour. Correspondent: Bruce Reed.

Does every digraph D have a vertex v such that

$$|N^{+}(v)| \leq |N^{++}(v)|,$$

where $N^{+}(v)$ is the (out-)neighbourhood of v and $N^{++}(v)$ the strict second neighbourhood, the set of vertices reachable by directed paths of length 2 but not by single arcs from v?

The truth of this would imply that of the Caccetta–Häggkvist conjecture, according to which a digraph on n vertices with in- and out-degrees at least $n/3$ contains a directed triangle.

Remark: Gregory Gutin has pointed out that a special case of this problem, known as Dean's Conjecture, was recently solved by Fisher [47].

Problem BCC16.13 (DM326): Book numbers of graphs. Proposed by E. Győri. Correspondent: E. Győri.

The *book* B_k is the graph consisting of k triangles sharing an edge. For a graph G, define $bn(G) = \max\{k : B_k \subseteq G\}$.

For $\frac{1}{2} < c < 1$, define

$$b(c) = \lim_{n\to\infty} \frac{1}{n} \min\{bn(G) : |V(G)| = n, d(x) \geq cn \forall x\}.$$

The problem is to determine $b(c)$. The intriguing conjecture about its value follows.

Let x be rational with $\frac{1}{2} < x < 1$. The "greedy representation" of x is given by

$$x = \frac{k_1 - 1}{k_2} \cdot \frac{k_2 - 1}{k_2} \cdots \frac{k_r - 1}{k_r},$$

where $k_i > (k_{i-1} - 1)^2$ for $i = 2, 3, \ldots, r$. (This representation is unique.) Then set

$$f(x) = \frac{k_1 - 2}{k_2} \cdot \frac{k_2 - 2}{k_2} \cdots \frac{k_r - 2}{k_r}.$$

Then f extends to a function on the real interval $(\frac{1}{2}, 1)$, continuous on every irrational. It is monotonic increasing left continuous but jumps at each rational.

Conjecture: $b(c) = f(c)$.

Remark: This is easy for $r = 1$, and is true (with a 20-page proof, see [45]) for $r = 2$.

Problem BCC16.14 (DM327): Automorphisms of lexicographic squares. Proposed by Gert Sabidussi. Correspondent: Geña Hahn.

Let G be a graph such that $G[G] \simeq G$, where $G[G]$ is the lexicographic product of G with itself.

Conjecture: There is an automorphism ϕ of $G[G]$ and vertices u, v, w, x, y of G such that $\phi(u,x) \in \{v\} \times V(G)$, $\phi(u,y) \in \{w\} \times V(G)$, and $v \neq w$.

Problem BCC16.15 (DM328): Simultaneous edge-colourings. Proposed by A. D. Keedwell. Correspondent: P. J. Cameron.

Suppose that $x_1, \ldots, x_m, y_1, \ldots, y_m$ are positive integers such that there exists a bipartite graph with vertex degrees $x_1, \ldots, x_m$ in one bipartite block and $y_1, \ldots, y_n$ in the other. (This is equivalent to asserting that the conditions of the Gale–Ryser theorem are satisfied.) Suppose further that all the x_i and y_j are greater than 1. Show that there is a bipartite graph having these vertex degrees, which has two proper edge-colourings such that

- for any vertex, the sets of colours appearing on edges at that vertex are the same in both colourings;
- no edge receives the same colour in both colourings.

Note: M. Mahdian, E. S. Mahmoodian, A. Saberi, M. R. Salavatipour and R. Tusserkani [77] have made a stronger conjecture, namely, that any bridgeless bipartite graph has a pair of colourings satisfying the above properties. They have shown that this conjecture is equivalent to the celebrated 'oriented cycle double cover conjecture' of Paul Seymour [96].

More recently (29 November 2000) I have learned that this problem has been solved by Rong Luo, Wen-An Zang and Cun-Quan Zhang [76].

Problem BCC16.16 (DM329): Abnormal strongly regular graphs. Proposed by P. J. Cameron and P. H. Fisher. Correspondent: P. J. Cameron.

Call a strongly regular graph Γ *abnormal* if it contains vertices x,y,z such that $x \not\sim y$, $x \not\sim z$, $y \sim z$, and $\Gamma(x) \cap \Gamma(y) = \Gamma(x) \cap \Gamma(z)$.

Problem: Does an abnormal strongly regular graph exist?

Remark: Such a graph must have $\lambda \geq \mu$, and not all the induced subgraphs on the non-neighbours of a vertex can be edge-regular. Any strongly regular graph with $\lambda > 0$ and $\mu = 1$ is abnormal; but no such graphs are known.

Problem BCC16.17 (DM330): A problem on Soicher's graph. Proposed by Bill Martin. Correspondent: Bill Martin.

This problem concerns a distance-regular graph Γ on 672 vertices, based on the punctured Golay code of length 22, constructed by Leonard Soicher [98] (see Problem BCC15.13). It turns out that Soicher's graph, as well as being P-polynomial (that is, distance-regular), is also Q-polynomial; indeed, it is one of only two "sporadic" P- and Q-polynomial graphs (that is, not having classical parameters) which are known to the proposer. (The other is the doubled Higman–Sims graph.)

Problems:

(a) Find a simple description of Γ in $\mathrm{PG}(2,4)$.

(b) Is there a Q-poset for Γ? (See below.)

Let Γ be a distance-regular graph of diameter d. Let $E_0, E_1, \ldots, E_d$ be the minimal idempotents in the Bose–Mesner algebra for Γ (in some order). The poset $(\mathcal{P}, \leq)$ of height d is a *Q-poset* for Γ if

(i) the dth level $\mathcal{P}_d$ of $\mathcal{P}$ is the vertex set of Γ;

(ii) for $0 \leq i \leq d$, the incidence matrix W_i between the ith and dth levels of $\mathcal{P}$ has constant row sums;

(iii)

$$\mathrm{rowspace}(E_i) \subseteq \mathrm{rowspace}(W_i) \subseteq \bigoplus_{j=0}^{i} \mathrm{rowspace}(E_j)$$

for $0 \leq i \leq d$.

Remark: The standard example: the Boolean lattice of subsets of an n-set, truncated to height d, is a Q-poset for the Johnson graph $J(n,d)$. If we require equality on the right-hand side of the inequality in (iii), and if $\mathrm{rank}(W_i) = |\mathcal{P}_i|$ (both of which occur in the standard example), then the Q-poset for Soicher's graph would satisfy

$$\begin{aligned}
|\mathcal{P}_0| &= 1,\\
|\mathcal{P}_1| &= 1+55 = 56,\\
|\mathcal{P}_2| &= 1+55+385 = 441,\\
|\mathcal{P}_3| &= 1+55+385+231 = 672.
\end{aligned}$$

Also, presumably, the poset must be constructed from the punctured Golay code (or from $\mathrm{PG}(2,4)$, if problem (a) is solved). Since M_{22} has no permutation action on 56 points, a construction from the punctured Golay code would require some "symmetry-breaking".

Problem BCC16.18 (DM331): The Canterbury Parades. Proposed by D. A. Preece. Correspondent: D. A. Preece.

The organisers of the Seventeenth British Combinatorial Conference are planning a series of parades to entertain the delegates. Seventy-six trombones will lead the parades, with one hundred and ten cornets close behind. Since the mediaeval streets of Canterbury are quite narrow, the trombonists can march four abreast, and the cornettists five abreast. It is required first that any three trombonists march in the same row in exactly one parade. This of course means constructing a resolvable 3-$(76,4,1)$ design. Such designs are known, and have 925 parallel classes, which means that the daily parades will last for about two and a half years.

The marching orders for the cornettists require a resolvable 3-$(110,5,1)$ design.

Problem: Construct such a design.

Editor's Note: The required design has 981 parallel classes. So, for the parades, it would suffice to find a "partial design" in which any three trombonists march together at most once, and there are 925 parallel classes.

In general, one could ask for solutions to the diophantine equation

$$\binom{v_1-1}{t_1-1}\Big/\binom{k_1-1}{t_1-1} = \binom{v_2-1}{t_2-1}\Big/\binom{k_2-1}{t_2-1},$$

and then ask for a solution to the corresponding parade problem.

Problem BCC16.19 (DM332): Semi-Latin squares which are partial linear spaces. Proposed by L. H. Soicher, R. A. Bailey and P. J. Cameron. Correspondent: R. A. Bailey.

An $(n \times n)/k$ semi-Latin square is an arrangement of nk letters in an $n \times n$ square, with k letters per cell, such that each letter occurs once in each row and once in each column. It is a *Trojan* square if it can be obtained by superimposing k mutually orthogonal $n \times n$ Latin squares. It is a *partial linear space* if no two letters occur together in the same cell more than once. Trojan squares are partial linear spaces.

Bailey [8] showed that when $k = n - 1$ then any semi-Latin square which is a partial linear space must be Trojan, and arises from an affine plane with two distinguished parallel classes of lines.

Problem: If $k = n - 2$, must any semi-Latin square which is a partial linear space be Trojan?

Editor's Note: Semi-Latin squares which are partial linear spaces are known as SOMAs. See also BCC12.1 and BCC13.21.

Recent exhaustive checking by Soicher [99] shows that the answer to the problem stated is affirmative when $n = 6$ (there are no SLS-PLS or Trojan squares) and when $n = 7$.

Problem BCC16.20 (DM333): Blocking-set-free configurations. Proposed by H. Gropp. Correspondent: H. Gropp.

Is there a 3-chromatic linear 3-regular 3-uniform hypergraph with 16 vertices and 16 hyperedges? (Equivalently, a configuration 16_3 with no blocking set.)

Remark: This is an update of problems from earlier British Combinatorial Conferences: see Problems BCC13.18 and BCC14.14. See also [54] for the background and current state of knowledge.)

In particular, in BCC14.14 the proposer asked whether there is a 15_3 with no blocking set. This has been resolved negatively by unpublished results of Kel'mans, Lomonosov and Kornerup.

Problem BCC16.21 (DM334): A non-Desarguesian configuration. Proposed by Jane W. Di Paola. Correspondent: Jane W. Di Paola.

Prove that a non-Desarguesian projective plane must contain *Martinetti's third configuration*. (The lines of the configuration are ABF, BCD, CAE, DEI, EFG, FDH, AGJ, BHJ, CIJ and GHI.)

Remark: Killgrove has shown that each of the three non-Desarguesian planes of order 9 contains this configuration.

Problem BCC16.22 (DM335): A matrix problem. Proposed by David Bedford and Roger Whitaker. Correspondent: David Bedford.

Let

$$X = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ \vdots & & & \vdots \\ B_{t1} & B_{t2} & \cdots & B_{tt} \end{pmatrix},$$

where each B_{ij} is a $n \times n$ matrix. Assume that $B_{ii} = nI$ for all i, and that, for $i \neq j$, B_{ij} has the following properties:

- entries on the leading diagonal are 1 and all other entries are 0, 1 or 2;
- B_{ij} has row and column sums n;
- $B_{ij}^{\top} = B_{ji}$ (and hence X is symmetric);
- $B_{ij} + B_{ji} = 2J$, where J is the $n \times n$ matrix with every entry 1.

Problem: Prove that X has nullity $t-1$. (It is easily seen that $t-1$ is a lower bound for the nullity.)

Problem BCC16.23 (DM336): Representing orthogonal matroids. Proposed by P. J. Cameron. Correspondent: P. J. Cameron.

Let V and W be vector spaces over a field F. Suppose that $v_1, \ldots, v_n \in V$ and $w_1, \ldots, w_n \in W$ satisfy

$$\sum_{i=0}^{n} v_i \otimes w_i = 0. \qquad (1)$$

Then the matroids M and M' on the ground set $\{1, \ldots, n\}$ represented by $(v_1, \ldots, v_n)$ and $(w_1, \ldots, w_n)$ are *orthogonal*, in the sense that any base of M is disjoint from some base of M' and *vice versa*.

Problem: Let M and M' be matroids on $\{1, \ldots, n\}$ which are both representable over a field F and are orthogonal in the above sense. Do there exist representations of M and M' (by $v_1, \ldots, v_n \in V$ and $w_1, \ldots, w_n \in W$) over F such that the displayed equation holds?

Problem BCC16.24 (DM337): Subsums of signed permutations. Proposed by J. Schönheim. Correspondent: J. Schönheim.

Given a sequence $a_1, a_2, \dots, a_n$, being a permutation of the first n positive integers, we say it has the *NZS property* if it is possible to assign $\pm$ to its members so that no subsequence of consecutive elements has a zero sum modulo $2n+1$.

(a) Which permutations of $\{1,2,\dots,n\}$ have the NZS property?

(b) Show that for every n there exists a latin square of order n with entries from $\{1,2,\dots,n\}$ such that each of its rows has the NZS property.

(c) Find for every n a sequence $a_1, a_2, \dots, a_n$, being a permutation of the first n positive integers, such that every cyclic permutation of $a_1, a_2, \dots, a_n$ has the NZS property.

Problem BCC16.25 (DM338): Erdős–Ko–Rado at the court of King Arthur. Proposed by Fred Holroyd. Correspondent: Fred Holroyd.

King Arthur has n knights, who have permanent places round the Table. They are to be arranged into sorties, of k knights each, such that:

(a) any two sorties intersect;

(b) because of courtly rivalries, any two knights in any sortie must sit at least d places apart round the Table (where $d \geq 2$; that is, $d = 2$ means adjacent knights cannot be in the same sortie).

Is it true that the Erdős–Ko–Rado theorem still holds, in the sense that to maximize the number of sorties, King Arthur should belong to all of them?

Remark: This is known to be true if n is sufficiently large (in terms of d and k) or if $kd \leq n < (k+1)d$.

Problem BCC16.26: The thrackle problem. Proposed by J. H. Conway. Correspondent: J. H. Conway.

A *thrackle* consists of a set of points in the plane called *spots*, and a set of differentiable simple curves called *paths*, such that

- the ends of each path are distinct spots, and it contains no other spot;
- any two paths must *hit* once and only once, that is, have just one common point, at which they must have different tangents;

- every spot is a hit.

Problem: Is there a thrackle with more paths than spots?

The proposer has offered \$1000 for this problem. For registered delegates of BCC16 only, the value of the prize is increased to £1000. For more information, see [105].

6 BCC17

BCC17 was held at the University of Kent at Canterbury, 12-16 July 1999. The contributed papers were published in *Discrete Mathematics* **231** (2001). The problems appear on pages 469–478 of the journal.

Problem BCC17.1 (DM343): Drawing configurations in the plane. Proposed by Harald Gropp. Correspondent: Harald Gropp.

The projective plane $\mathrm{PG}(2,2)$ has a familiar drawing in the euclidean plane with six of its seven lines drawn as Euclidean straight lines. What is the maximum number of straight lines in a Euclidean drawing of $\mathrm{PG}(2,3)$? (A drawing with eight straight lines is known.)

Problem BCC17.2 (DM344): Nested BIBDs. Proposed by D. A. Preece and R. A. Bailey. Correspondent: R. A. Bailey.

Let Δ be a balanced incomplete block design (BIBD) on v letters, having b blocks of size k. Let Γ be a BIBD on the same v letters having bs blocks of size k/s (with $s > 1$), its blocks obtained by splitting the blocks of Δ into s subblocks. Then Γ is *nested* in Δ.

(a) Find examples with $\mathrm{Aut}(\Delta) = \mathrm{Aut}(\Gamma) = 1$.

(b) Do almost all pairs of nested BIBDs satisfy this?

Problem BCC17.3 (DM345): Intersecting families and $S(4,7,23)$. Proposed by Peter Rowlinson. Correspondent: Peter Rowlinson.

The family $\mathcal{F}$ of 7-subsets of a 23-set has the property that any two members of $\mathcal{F}$ intersect in 1 or 3 elements. (Hence $|\mathcal{F}| \leq 253$.) How large must $|\mathcal{F}|$ be to guarantee that $\mathcal{F}$ can be embedded in $S(4,7,23)$?

Problem BCC17.4 (DM346): An extremal problem related to biplanes. Proposed by Gregory Gutin. Correspondent: P. J. Cameron.

Given n, what is the smallest m such that there exist m subsets (called blocks) of the point set $\{1,\dots,n\}$ such that

(a) any two points lie in *at least* two blocks,

(b) any two blocks meet in *at most* two points?

Remark: It is known that $n \le m \le (2+o(1))n$; the lower bound comes from a simple counting argument, and the upper bound is obtained by taking blocks to be the translates of D and $-D$, where D is a planar difference set in C_n, with $n = q^2+q+1$. For more details, see [25].

Problem BCC17.5 (DM347): Some 1-factorizations. Proposed by Chris Rodger. Correspondent: Chris Rodger.

Let G be the multigraph whose vertex set is $\mathbb{Z}_{2n}$, the integers modulo $2n$, in which i and j are joined by two edges if $j = i+n$ and one edge otherwise. Give an easy way to find a 1-factorization of G in which each of the $2n$ 1-factors contains one of the doubled edges $\{i, i+n\}$.

This 1-factorization is equivalent to a symmetric Latin square with holes of size 2, so can be constructed reasonably easily using design-theoretic techniques.

Editor's note: There is a simple solution when n is odd. Take the vertices to be those of a regular $2n$-gon, the doubled edges being the long diagonals. Now let one 1-factor containing a long diagonal $\{i, i+n\}$ contain all edges and short diagonals parallel to $\{i, i+n\}$, and the other contain all short diagonals perpendicular to $\{i, i+n\}$.

A solution for $n \equiv 0 \pmod 4$ is given by Bailey and Monod [10]; but this perhaps does not qualify as "easy".

Problem BCC17.6 (DM348): Unions of random matchings. Proposed by Nick Wormald. Correspondent: Nick Wormald.

Choose k perfect matchings of the complete graph K_n (for n even) at random. Denote them by $M_1,\dots,M_k$. Let $p_k(n)$ denote the probability that for every $i \ne j$, the union of M_i and M_j forms a Hamilton cycle. How does this probability behave as $n \to \infty$ with k fixed? In particular, is it true that

$$p_k(n) \sim (p_2(n))^{\binom{k}{2}}?$$

Remark: The answer to the last question is known to be "yes" for $k = 3$ (shown in joint work of the proposer and J. H. Kim [65]).

Problem BCC17.7 (DM349): Edge-colouring nearly complete graphs. Proposed by N. Zagaglia Salvi. Correspondent: N. Zagaglia Salvi.

Let G be a graph obtained from K_n, where $n = 2t+1$, by deleting any t edges. Let α be a proper χ'-edge-colouring of G, where $\chi' = \chi'(G) = n-1$.

Problems: (a) Does there exist a subgraph of G having $n-1$ edges and maximum degree $\Delta = 2$, whose edges all have different colours in the colouring α?

(b) For every edge $e \in E(G)$, does there exist a subgraph of G containing e which has $n-2$ edges and has maximum degree $\Delta = 2$, whose edges all have different colours in the colouring α?

Problem BCC17.8 (DM350): Caterpillar-arboricity of planar graphs. Proposed by Y. Roditty. Correspondent: Y. Roditty.

The *arboricity* $a(G)$ of a graph G is defined to be the smallest number of forests containing all the edges of G. In a similar way, the *linear arboricity* $la(G)$ (resp., *star arboricity* $sa(G)$, *caterpillar arboricity* $ca(G)$) of G is the smallest number of forests containing all edges of G such that each component of each forest is a path (resp., a star, a caterpillar). (A *caterpillar* is a tree with the property that removal of all the end vertices and the edges containing them yields a path.)

Nash-Williams [87] proved that any planar graph G satisfies $a(G) \leq 3$. Hakimi, Mitcham and Schmeichel [57] showed that a planar graph satisfies $sa(G) \leq 5$.

Conjecture: A planar graph G satisfies $ca(G) \leq 4$.

Problem BCC17.9 (DM351): Binding functions for graphs. Proposed by Ingo Schiermeyer and Bert Randerath. Correspondent: Ingo Schiermeyer.

As introduced by Gyárfás [55], a family $\mathcal{G}$ of graphs is called χ-*bound* with χ-*binding function* f if $\chi(G') \leq f(\omega(G'))$ holds whenever G' is an induced subgraph of $G \in \mathcal{G}$. (Here as usual $\omega(G')$ and $\chi(G')$ are the clique number and chromatic number of G'.)

Let $\mathcal{G}^I(3,4)$ denote the class of graphs whose induced cycles have length 3 or 4 only.

Problem: Determine a χ-binding function for $\mathcal{G}^I(3,4)$.

Remark: The authors [90] have shown that this class does not have a linear χ-binding function.

Problem BCC17.10 (DM352): Group analogues of graph problems. Proposed by Frank Harary. Correspondent: Frank Harary.

Find and solve problems about finite groups (or finite abelian groups) motivated by results in graph theory.

For example, Harary and Hawthorn [58] found the minimum number of elements in a set $S \subseteq A \setminus \{0\}$ such that $A \setminus S$ has no subgroup isomorphic to B, where A and B are finite abelian groups; this is an analogue of Turán's Theorem for graphs.

Problem BCC17.11 (DM353): Neighbourhood-symmetric graphs. Proposed by Dalibor Fronček. Correspondent: Dalibor Fronček.

The *neighbourhood* of a vertex x in a graph G, denoted $N_G(x)$, is the *subgraph* of G induced on the set of all neighbours of x. We say that G has *constant neighbourhood* H if $N_G(x) \cong H$ for all $x \in V(G)$. We say that G is a *neighbourhood-symmetric graph* (or NSG) if $N_G(x) \cong N_{\overline{G}}(x) \cong H$, for some H, and all $x \in V(G)$. Clearly any vertex-transitive self-complementary graph is a NSG.

Problem: Construct a counterexample to the converse assertion; that is, find a NSG which fails to be vertex-transitive and self-complentary.

Problem BCC17.12 (DM354): Semiregular automorphisms. Proposed by Peter Cameron and John Sheehan. Correspondent: P. J. Cameron.

Marušič and Scapellato [79] proved that a vertex-transitive connected cubic simple graph has a non-trivial semiregular automorphism (one for which all cycles have the same length). Is it true that there exists such an automorphism having order at least $f(n)$, where n is the number of vertices and f is a function for which $f(n) \to \infty$ as $n \to \infty$? Easy examples show that $f(n)$ cannot exceed $O(n^{1/3})$.

Editor's note: The proposers have recently shown that there is a semiregular automorphism of order greater than 2.

Problem BCC17.13 (DM355): Regular graphs admitting a given group. Proposed by Peter J. Cameron. Correspondent: Peter J. Cameron.

It is known (see [22]) that, for any finite group Γ, there exists a rational number $a(\Gamma) \in [0,1]$ such that, if G denotes a random graph on the vertex set $\{1,\ldots,n\}$ (with all graphs equally likely), then

$$\mathrm{Prob}(\mathrm{Aut}(G) = \Gamma \mid \mathrm{Aut}(G) \geq \Gamma) \to a(\Gamma) \text{ as } n \to \infty.$$

Does a similar result hold for other random graph models, in particular for random regular graphs of degree $d > 2$ (as described by Nick Wormald [106] at the conference)?

Problem BCC17.14 (DM356): Symmetry groups of boolean functions. Proposed by Andrzej Kisielewicz. Correspondent: Andrzej Kisielewicz.

Let f be a *k-valued boolean function* of n variables, that is, a function from $\{0,1\}^n$ to $\{0,1,\ldots,k-1\}$. If $k=2$, we simply call f a *boolean function*. The *symmetry group $G(f)$* of f is the group of all permutations $\sigma \in S_n$ such that

$$f(x_{\sigma(1)},\ldots,x_{\sigma(n)}) = f(x_1,\ldots,x_n)$$

for all $(x_1,\ldots,x_n) \in \{0,1\}^n$.

We say that a subgroup G of S_n is *k-representable* if $G = G(f)$ for some k-valued boolean function f. Clote and Kranakis [34] proved that, if G is k-representable for some $k \geq 2$, then G is 2-representable. However, the Klein group

$$V_4 = \langle (1\,2)(3\,4),(1\,3)(2\,4)\rangle \leq S_4$$

is a counterexample: it is 3-representable but not 2-representable. The proof in [34] has a gap, which does not seem to be fixable. For more information see [66].

Problem: Are there any other counterexamples?

Problem BCC17.15 (DM357): Non-zero-sum sequences. Proposed by J. Schönheim. Correspondent: J. Schönheim.

Let $a_i \in \mathbb{Z}_{2m+1}$ (the integers modulo $2m+1$) for $i=1,\ldots,t$. We say that the sequence $a = (a_1,\ldots,a_t)$ has the *k-NZS property* if no subsequence of k or fewer consecutive terms has sum zero (mod $2m+1$).

In these problems, $a = (a_1,\ldots,a_m)$ denotes a permutation of $(1,\ldots,m)$, and $\varepsilon = (\varepsilon_1,\ldots,\varepsilon_m)$ a sequence of signs ($\varepsilon_i = \pm 1$ for all i).

It is known that there exist such sequences a,ε such that the sequence $(\varepsilon_1 a_1,\ldots,\varepsilon_m a_m)$ has sum zero (mod $2m+1$) but has the $(m-1)$-NZS property.

Problem: Given $b_1,\ldots,b_m$, show that there exist $c_1,\ldots,c_m$ such that

(a) $(c_1,\ldots,c_m)$ is a permutation of $(\varepsilon'_1 b_1,\ldots,\varepsilon'_m b_m)$ for some choice of signs $\varepsilon' = (\varepsilon'_1.\ldots,\varepsilon'_m)$;

(b) each row in the array

$$\begin{array}{ccccc} b_1 & b_2 & \ldots & b_{m-1} & c_m \\ b_2 & b_3 & \ldots & b_m & c_1 \\ b_3 & b_4 & \ldots & b_1 & c_2 \\ \vdots & & & & \\ b_m & b_1 & \ldots & b_{m-2} & c_{m-1} \end{array}$$

is m-NZS.

Problem: Prove that given ε, there exists a such that the sequence εa has the m-NZS property.

Problem BCC17.16 (DM358): Acceptable cross-sections. Proposed by Nigel Martin. Correspondent: Nigel Martin.

A *cross-section* of a sequence $(S_1,\ldots,S_k)$ of sets of integers is a sequence $(x_1,\ldots,x_k)$ with $x_i \in S_i$ for $i=1,\ldots,k$. A cross-section is *acceptable* if for all i,j with $i \neq j$,

$$x_j - x_i \not\equiv j-i \pmod{k}.$$

Fix $p=2r+1$, and consider the sets $S_1,\ldots,S_p$ given by

$$\begin{aligned} S_i &= \{j : i \leq j \leq r+i-1\} \quad \text{for} \quad 1 \leq i \leq r, \\ S_{r+1} &= \{j : 1 \leq j \leq r\}, \\ S_{r+i+1} &= \{j : i \leq j \leq r+i\} \quad \text{for} \quad 1 \leq i \leq r-1, \\ S_{2r+1} &= \{j : r \leq j \leq r-1\} \end{aligned}$$

Find $p-2$ acceptable cross-sections for this sequence so that, in aggregate, every number in the range $\{1,\ldots,p-2\}$ occurs $p+1$ times.

Solutions are known for $p=2^n \pm 1$, $p=6\cdot 2^n-1$ and finitely many other values.

Editor's Note: This problem has been solved by Richard Stong.

Problem BCC17.17 (DM359): Evaluating inversion numbers. Proposed by Timothy R. Walsh. Correspondent: Timothy R. Walsh.

The number $M(n,r)$ of permutations of $\{1,2,\ldots,n\}$ with r inversions is the coefficient of x^r in

$$(1+x)(1+x+x^2)\cdots(1+x+x^2+\cdots+x^{n-1}).$$

To find a single value of $M(n,r)$ by evaluating $M(n',r')$ for all $n' \leq n$ and $r' \leq r$ takes $O(nr)=O(n^3)$ arithmetic operations.

Problem: Find a non-recursive formula for $M(n,r)$ which can be evaluated in at most the same time.

Donald E. Knuth [70], page 16 gave a simple formula in the case $r \leq n$. The proposer has given a monstrously complicated formula for the general case.

Problem BCC17.18 (DM360): Sets of permutations with given minimum distance. Proposed by Wendy Myrvold. Correspondent: Wendy Myrvold.

What is the largest number of permutations of n symbols in a set with the property that any two agree in at most r columns? This number is at most $n!/(n-r-1)!$, with equality if and only if the set is sharply $(r+1)$-transitive.

For further details on this problem, see Section 5 of the survey [26].

Problem BCC17.19 (DM361): Covering radius and Tutte polynomial. Proposed by Carrie G. Rutherford, Fuad Shareef. Correspondent: Peter J. Cameron.

Associated with any matrix A over a field F, there is a matroid representable over F (whose elements are the columns of A, and in which dependence is linear dependence) and a linear code (spanned by the rows of F. Greene [53] showed that the weight enumerator of the code is a specialisation of the Tutte polynomial of the matroid.

Do there exist two binary linear codes which have the same Tutte polynomials but different covering radii? (There are codes with the same weight enumerators but different covering radii.)

Editor's Note: At the conference, the following problem was presented:

Problem BCC17.20: Derangements in the alternating group. Proposed by Ömer Eceğlioğlu. Correspondent: Ömer Eceğlioğlu.

Let d_n be the number of derangements (fixed-point-free permutations) in the symmetric group S_n, and a_n the number of derangements in the alternating group A_n. It is known that

$$a_n = \binom{n}{2} d_{n-2} + (-1)^{n-1}(n-1).$$

Problem: Find a bijective proof of this fact.

The problem is equivalent to the statement that the numbers $d_n^+ = a_n$ and d_n^- of derangements which are even and odd permutations satisfy

$$d_n^+ - d_n^- = (-1)^{n-1}(n-1).$$

This was subsequently solved by Robin Chapman. His elegant solution follows.

Let D_n the be set of derangements. We can split this set into $n-1$ equal parts according to where the permutation sends n. We look therefore only at $D'_n = \{\sigma \in D_n : \sigma(n) = n-1\}$. Define a sign-reversing involution on the set $D'_n \setminus \{(1\ 2\ 3\ \dots\ n-2\ n\ n-1)\}$. This will give us what we want.

For $\sigma \in D'_n$ let $a(\sigma)$ be the least number a with $\sigma(a) < a$. Then $a(\sigma) < n-1$ with the sole exception $\sigma = (1\ 2\ 3\ \dots\ n-2\ n\ n-1)$. For all $\sigma \in D'_n$ apart from this one permutation, let $f(\sigma) = (a(\sigma)\ n) \circ \sigma$.

It suffices to show that $\tau = f(\sigma)$ satisfies $\tau \in D'_n$ and $a(\tau) = a(\sigma)$.

First of all $\sigma(n) = n-1$ which is not in $\{a, n\}$, so $\tau(n) = n-1$. Suppose $\tau(j) = j$ for some j. Then $\tau(j) \neq \sigma(j)$, so that $\sigma(j) = a$ or n and $\tau(j) = j = n$ or a respectively. But $\sigma(a) < a$, so $j = a$ and $\sigma(j) = n$ is impossible. Also $\tau(n) = n-1 > a$, so that $j = n$ and $\sigma(j) = a$ is impossible. Hence τ lies in D'_n.

If $j < a$, then $\sigma(j) > j$ and $\tau(j)$ equals one of $\sigma(j)$, a and n, all of which exceed j. But $\sigma(a) < a < n$ and so $\tau(a) = \sigma(a) < a$. Hence $a(\tau) = a$.

7 BCC18

BCC18 was held at the University of Sussex, 2-6 July 2001.

Problem BCC18.1: Freese–Nation numbers of posets. Proposed by D. H. Fremlin and D. B. Penman. Correspondent: D. B. Penman.

Let $(P, \preceq)$ be a poset. A function $f : P \mapsto \mathcal{P}P$ (where $\mathcal{P}P$ is the power set of P) is a *Freese–Nation function* if, whenever $p \preceq q$, we have

$$f(p) \cap f(q) \cap [p, q] \neq \emptyset.$$

The *Freese–Nation number* $\mathrm{FN}(P)$ is the smallest r for which there is a Freese–Nation function f with $|f(p)| < r$ for all $p \in P$. Observe that $p \in f(p)$ for all $p \in P$.

For example,

1. if P is an antichain, then $\mathrm{FN}(P) = 2$;
2. if P is an n-element chain, then $\mathrm{FN}(P) = 2 + \lfloor \log_2 n \rfloor$;
3. if $P = A \cup B$ with $|A| = 2r-5$, $|B| = 2r-6$, and $a \preceq b$ for all $a \in A$, $b \in B$, then $\mathrm{FN}(P) = r$;
4. If P is selected from the uniform distribution on n-element posets, then $\mathrm{FN}(P) = (n/8)(1 + o(1)$ with high probability.

Problem: Find

$$\lim_{m\to\infty} (\mathrm{FN}(\mathcal{P}m))^{1/m},$$

where m denotes an m-element set. (It is known that the limit exists and lies in the interval $[2/\sqrt{3}, \sqrt[3]{3}] \approx [1.1547, 1.4422]$.)

Problem BCC18.2: Matching roots of vertex-transitive graphs. Proposed by Bojan Mohar. Correspondent: Bojan Mohar.

Let $p(G,k)$ be the number of matchings of the graph G with k edges. Then the *matching polynomial* of G is

$$\mu(G,x) = \sum_{k=0}^{\lfloor n/2 \rfloor} (-1)^k p(G,k) x^{n-2k}.$$

It is known that $\mu(G,k)$ has only real roots.

Conjecture: For every integer r there exists a connected vertex-transitive graph whose matching polynomial has a root of multiplicity at least r.

Even examples of vertex-transitive graphs with at least one non-simple root would be of great interest, since such graphs cannot contain a Hamiltonian path (see [51, 59]).

Editor's note: This was the proposer's "Problem of the month" for July 2001: see `http://www.fmf.uni-lj.si/~mohar/Problems.html`

Problem BCC18.3: Strongly distance-regular graphs. Proposed by M. A. Fiol. Correspondent: M. A. Fiol.

For the definition of a distance-regular graph and related concepts, we refer to Brouwer *et al.* [19].

A graph Γ with diameter d is called *strongly distance-regular* if Γ is distance-regular and the distance-d graph Γ_d (in which vertices are adjacent if they have distance d in Γ) is strongly regular. Examples include

1. any strongly regular graph;
2. any distance-regular graph with $d = 3$ and third-largest eigenvalue -1;
3. any antipodal distance-regular graph.

Problem: Prove or disprove that these examples exhaust all possibilities.

Problem BCC18.4: Some configurations in polar spaces. Proposed by Harm Pralle. Correspondent: Harm Pralle.

For which polar spaces Π of rank 3, other than the Klein quadric, does there exist a set H of planes such that

(i) there exists a unique plane $\delta \in H$ such that any plane of Π intersecting δ in a line belongs to H, and

(ii) every line of Π not contained in δ is covered uniquely by a plane of H?

The only known example for H lives in the symplectic variety $S_5(\mathbb{R})$ in $\mathrm{PG}(5,\mathbb{R})$; it is a hyperplane of the dual of $S_5(\mathbb{R})$ arising from an embedding in $\mathrm{PG}(13,\mathbb{R})$. (All examples in the Klein quadric are obtained by taking δ to be a plane and including also all the planes of the opposite ruling.)

Problem BCC18.5: Projective space analogues of Steiner systems. Proposed by "Folklore" (possibly Ph. Delsarte). Correspondent: Peter J. Cameron.

Does there exist a collection S of planes in the projective space $\mathrm{PG}(n,q)$, where $n > 2$, such that any line lies in a unique member of S? (This would be the analogue for projective spaces of a Steiner triple system.) No examples are known.

One can easily define analogues of arbitrary t-designs in projective spaces (probably Delsarte [37] was the first to do so), but very few examples are known. However, infinite examples exist in great profusion!

Problem BCC18.6: "Problem 6". Proposed by Harald Gropp. Correspondent: Harald Gropp.

Is there a bipartite 6-regular graph with 66 vertices having girth 6?

Equivalently, is there a 33_6 configuration? (This is a configuration with 33 points and 33 lines, each point on 6 lines and each line containing 6 points, such that two points lie on at most one line.)

Problem BCC18.7: Multiplication group of a Latin square. Proposed by Aleš Drápal. Correspondent: Aleš Drápal.

Consider a Latin square L of order n whose first row and column are normalised to have the entries $1,\ldots,n$ in order. Each row and column of L is a permutation of $\{1,\ldots,n\}$; the group generated by these permutations is the *multiplication group* of L, denoted by $M(L)$.

Given $k \geq 3$ and a prime power q, does there exist a Latin square L such that

$$\mathrm{PSL}(k,q) \leq M(L) \leq \mathrm{P\Gamma L}(k,q)?$$

The proposer has shown recently [41, 42] that, if $k = 2$, there is only one such square L, with $M(L) = \mathrm{P\Gamma L}(k,q) = S_5$.

For the next two problems, we introduce a Markov chain method for choosing Latin squares uniformly at random, due to Jacobson and Matthews [63].

We represent a Latin square of order n by a function $f : N^3 \to \{0,1\}$ (where $N = \{1,\ldots,n\}$) satisfying

$$\sum_{x \in N} f(x,y,z) = 1$$

for given $y,z \in N$, and two similar equations for the other coordinates. We allow also *improper Latin squares*, which are functions satisfying these constraints but which take the value -1 exactly once. Now to take one step in the Markov chain starting at a function f, do the following:

(a) If f is proper, choose any (x,y,z) with $f(x,y,z) = 0$; if f is improper, use the unique triple with $f(x,y,z) = -1$.

(b) Let $x',y',z' \in N$ satisfy

$$f(x',y,z) = f(x,y',z) = f(x,y,z') = 1.$$

(If f is proper, these points are unique; if f is improper, there are two choices for each of them.)

(c) Now increase the value of f by one on the triples (x,y,z), (x,y',z'), (x',y,z') and (x',y',z), and decrease it by one on the triples (x',y,z), (x,y',z), (x,y,z') and (x',y',z'). We obtain another proper or improper Latin square, according as $f(x',y',z') = 1$ or 0.

Jacobson and Matthews show that the limiting distribution gives the same probability to each Latin square.

Problem BCC18.8: Choosing Latin squares uniformly at random. Proposed by M. T. Jacobson and P. Matthews; J. Møller; J. Besag. Correspondent: R. A. Bailey.

Problem: How fast does the Jacobson–Matthews Markov chain converge to the uniform distribution?

Problem BCC18.9: A Markov chain for Steiner triple systems. Proposed by Peter J. Cameron. Correspondent: Peter J. Cameron.

A slight modification of the method of Jacobson and Matthews should work for Steiner triple systems. We simply replace "ordered triples" by "unordered triples of distinct elements" in the definition; then a STS is a function from unordered triples to $\{0,1\}$ which satisfies

$$\sum_{z \neq x,y} f(\{x,y,z\}) = 1$$

for all distinct points x,y, and an improper STS is allowed to take the value -1 exactly once. Now the moves are defined as before. However, before we know that the limiting distribution is uniform, we have to solve the following

Problem: Is the chain connected? That is, is it possible to get from any STS to any other by a sequence of moves?

Problem BCC18.10: Perfect Steiner triple systems. Proposed by M. J. Grannell and T. S. Griggs. Correspondent: T. S. Griggs.

Let $S = (V,B)$ be a Steiner triple system of order v, and let a and b be any two points, and c the third point of the block containing them. Define a graph G_{ab} as follows: the vertex set is $V \setminus \{a,b,c\}$, and two vertices x and y are adjacent if and only if either $\{a,x,y\} \in B$ or $\{b,x,y\} \in B$. Clearly G_{ab} is a union of disjoint even cycles. If G_{ab} is a single cycle for *all* choices of $a,b \in V$, then S is said to be *perfect*.

Perfect STS of orders 7, 9, 25 and 33 have been known for some time. More recently Grannell, Griggs and Murphy [52] added nine new values to the list of orders:

79, 139, 367, 811, 1531, 25771, 50923, 61339, 69991.

These are all primes of the form $12s+7$.

Problem: What number-theroretic property distinguishes these nine primes from the other primes of this form less than 100000 (where the search terminated)?

The next two problems refer to circular chromatic number, which is defined as follows. For a hypergraph H, and positive integers p,q with $2q \leq p$, we define a (p,q)*-colouring* to be a function $c : V(H) \to \{0,1,\ldots,p-1\}$ such that each edge e of H contains two vertices a and b with $q \leq |c(x)-c(y)| \leq p-q$. The *circular chromatic number* of H, written $\chi_c(H)$, is the infimum of the set of values p/q for which there exists a (p,q)-colouring of H. (We can replace "inf" by "min" here.) Alternatively, it is the smallest circumference of a circle S such that the vertices of the graphs can be mapped to points of S such that adjacent points are at distance at least 1. The definition of circular chromatic number of a graph is just the specialisation of this definition.

Since every p-colouring is a $(p,1)$-colouring, we have $\chi_c(H) \le \chi(H)$, where $\chi(H)$ is the chromatic number of H.

See Zhu [111] for a survey, and also the paper by Mohar [82] presented at the meeting.

Problem BCC18.11: Circular chromatic number of Steiner triple systems. Proposed by Changiz Eslahchi, Arash Rafiey. Correspondent: Changiz Eslahchi.

Conjecture: For every Steiner triple system S of order at least 13, we have $\chi_c(S) = \chi(S)$.

Editor's note: The conjecture is false for order 7. I am grateful to Fred Holroyd for pointing out to me that the usual cyclic representation of the STS of order 7 shows that $\chi_c(S) \le 7/3$, while of course $\chi(S) = 3$.

Problem BCC18.12: Bounding the circular chromatic number of a graph. Proposed by Bojan Mohar. Correspondent: Bojan Mohar.

Let $P_G(x)$ be the chromatic polynomial of the graph G and let k be the chromatic number of G. Let $c_0 \le k$ be the largest real number such that $P_G(c_0) = k!$.

Problem: Is it true that $\chi_c(G) \le c_0$, where $\chi_c(G)$ is the circular chromatic number of G?

Problem BCC18.13: Two list colouring conjectures. Proposed by S. Akbari, V. S. Mirrokni, B. S. Sadjad. Correspondent: S. Akbari.

1. *A list edge-colouring conjecture.* Let G be a graph with m edges and maximum degree $\Delta \ge 2$. Suppose that $L = \{L_1, \ldots, L_m\}$ is an assignment of lists of colours to the edges of G such that $|L_i| = \Delta$ for $i = 1, \ldots, m$. Show that G is *not* uniquely L-colourable.

This is known to be true if G is not regular, or if G is regular and bipartite (see [15]).

2. *A list vertex-colouring conjecture.* Suppose that G is a graph and $f : V(G) \to \mathbb{N}$ is a function, where $\mathbb{N}$ is the set of natural numbers. Let L be a list assignment to the vertices of G, such that $|L_v| = f(v)$ for any $v \in V(G)$, and assume that G is uniquely L-colourable. Suppose that G is a maximal uniquely f-colorable graph (that is, for any list assignment L' of G, if $f(v) \le |L'_v|$ for all $v \in V(G)$ and there exists a vertex v_0 such that $f(v_0) < |L'_{v_0}|$, then G is not uniquely L'-colorable). Then G is f-choosable.

Problem BCC18.14: Colouring and degeneracy of random graphs. Proposed by Bojan Mohar. Correspondent: Bojan Mohar.

Here $\mathcal{G}_{n,p}$ denotes the random graph model in which edges are selected from the n-vertex set independently with probability p (see Molloy's paper [83] presented at the conference). A graph is k-degenerate if every induced subgraph has a vertex of degree smaller than k. Clearly a k-degenerate graph is k-colourable. A *k-core* of a graph is an induced subgraph with minimum degree at least k.

Let $p = p(n,k)$ be the smallest probability such that almost no graphs in $\mathcal{G}_{n,p}$ are $(k \log k)$-degenerate.

Conjecture: Almost all graphs in $\mathcal{G}_{n,p}$ have chromatic number at least k. (In other words, the threshold for a $(k \log k)$-core is at least that for k-colourability.)

Problem BCC18.15: Odd holes in planar graphs. Proposed by Colin McDiarmid. Correspondent: Colin McDiarmid.

An *odd hole* in a graph is an induced subgraph which is an odd circuit of length at least 5.

Does every planar graph have 3-colouring (not necessarily proper) of the vertices such that every odd hole receives all three colours?

This question is related to measuring how imperfect a planar graph can be.

Problem BCC18.16: Chord diagrams and Vassiliev invariants. Proposed by Leonid Plachta. Correspondent: Leonid Plachta.

The following combinatorial problem arises in the study of Vassiliev knot invariants. To formulate it let us first recall that each n-singular knot (C^1-immersion of S^1 into $\mathbb{R}^3$) with exactly n double transverse points (called singularities) can be represented (though not uniquely) by its *chord diagram* (for short, CD), in which the preimages of each singular point in S^1 are the endpoints of a chord in the CD.

Let $\mathcal{K}$ denote the set of knots in $\mathbb{R}^3$. Any isotopy invariant of knots $v\colon \mathcal{K} \to \mathbb{Q}$ can be extended in a natural way to the set $\mathcal{L}$ of singular knots with a finite number of singularities (see, for example, [11]). An isotopy invariant $v\colon \mathcal{L} \to \mathbb{Q}$ is called a *Vassiliev invariant* of order n if v vanishes on any $(n+1)$-singular knot and n is the smallest number with this property. It turns out (see [11]) that any Vassiliev invariant v of order n has equal values on all singular knots having the same CDs with n chords.

Let D_n denote the set of chord diagrams with n chords, the CDs being considered up to the obvious equivalence relation, and let $\mathrm{span}(D_n)$ be the vector space over $\mathbb{Q}$ generated by D_n. It follows any $\mathbb{Q}$-valued Vassiliev invariant v of order n determines a function $w(v)\colon D_n \to \mathbb{Q}$ satisfying the axioms 1T (framing independence) and 4T (the four term relation) described, for example, in [11]. Such a function is called a *weight system* of degree n. In other words, a weight system of degree n is an element of the dual space of the vector space

$$\mathcal{A}_n = \mathrm{span}(D_n)/\mathrm{span}(\{4\mathrm{T}\text{ and }1\mathrm{T relations}\}).$$

For any $D \in D_n$, let $G(D)$ denote the *intersection graph* (or *interplay graph*, in the terminology of [1]) of D. Note that not every abstract intersection graph with n vertices is realizable as an intersection graph of some chord diagram of order n. Rosenstiehl's theorem characrerizes the class of all realizable abstract intersection graphs (see [1]).

The Intersection Graph Conjecture, formulated by Chmutov *et al.* [32], asserts that a weight system $w\colon D_n \to \mathbb{Q}$ has equal values on any two chord diagrams with the same intersection graphs, so its values on CDs are determined uniquely by their intersection graphs. They proved the conjecture in the case when the intersection graphs of chord diagrams are trees. It follows that the conjecture is true if the intersection graphs are forests. Recently B. Mellor [80] showed that the conjecture is true for chord diagrams whose intersection graphs have exactly one loop.

T. Q. T. Le showed however that, in general, the conjecture is false, since it implies that Vassiliev knot invariants cannot detect mutation, contradicting the Morton/Cromwell examples. More precisely, Morton and Cromwell [84] showed that there exists a framed Vassiliev invariant v of degree 11 with values in $\mathbb{Z}[u]$ which takes different values on Kinoshita-Teresaka/Conway mutants. This implies that there exists a (framing independent) $\mathbb{Q}$-valued Vassiliev invariant of order 11 distinguishing both the mutants (see [102]). This example yields two singular knots representing by CDs D_1 and D_2 of order 11, with the same intersection graphs $G(D_1)$ and $G(D_2)$, and such that $[D_1] \neq [D_2]$ in $\mathcal{A}_{11}$.

Problem: Describe the class of all (realizable) intersection graphs for which the Intersection Graph Conjecture is true.

Problem BCC18.17: Fragmentability of graphs of bounded degree. Proposed by Keith Edwards, Graham Farr. Correspondent: Graham Farr.

Let C be a positive integer and α a real number in $(0,1)$. A graph G on n vertices is (C,α)-*fragmentable* if there exists a set X of at most αn vertices such that each component of $G-X$ has at most C vertices.

Problem: Does there exist $\alpha < 1$ and a sequence $C_1, C_2, \ldots$ of constants such that every graph G of maximum degree Δ is (C_Δ, α)-fragmentable?

It is known that such an α must be at least $1/2$: see [43].

Problem BCC18.18: Monotone paths in edge-ordered graphs. Proposed by Yehuda Roditty. Correspondent: Yehuda Roditty.

An *edge-ordered graph* is an ordered pair (G, f), where $G = G(V,E)$ is a finite undirected simple graph and f is a bijection from $E(G)$ to $\{1,2,\ldots,|E(G)|\}$, called an *edge-ordering* of G. A *monotone path of length k* in (G, f) is a simple path $P_{k+1} : v_1, v_2 \ldots, v_{k+1}$ in G such that the values $f((v_i, v_{i+1}))$, for $i = 1,2,\ldots,k-1$, are strictly monotonic (either increasing or decreasing). All definitions and updated results can be found in [93].

Given a graph G, denote by $\alpha(G)$ the minimum (over all edge orderings of G) of the maximum length of a monotone path.

Problems:

1. Prove that $\alpha(K_n) = (\frac{1}{2} + o(1))n$. (The right-hand side is known to be an upper bound for $\alpha(K_n)$.)
2. Determine $\alpha(G)$ for G a planar graph. (It is known that $5 \leq \alpha(G) \leq 9$, and if G is bipartite then $4 \leq \alpha(G) \leq 6$).

Problem BCC18.19: Decomposing complete multipartite graphs. Proposed by Keith Edwards. Correspondent: Keith Edwards.

A graph H *decomposes* a graph G if there is a set S of subgraphs of G, each isomorphic to H, such that each edge of G is contained in exactly one of the graphs in S.

Problem: Is it true that, for any λ-partite graph H, there is an integer n such that H decomposes the complete λ-partite graph with all parts of size n?

The answer is "yes" for $\lambda = 2$ and $\lambda = 3$.

Problem BCC18.20: Graphs isomorphic to their neighbourhoods and non-neighbourhoods. Proposed by Anthony Bonato. Correspondent: Anthony Bonato.

Let $N(x)$ and $N^c(x)$ denote the sets of neighbours and non-neighbours of the vertex x of a graph G, respectively. We say that G has *property* (N) if, for every vertex x, the subgraph induced by $N(x)$ is isomorphic to G; property (N^c) is defined similarly.

Problem Which countable simple graphs have *both* property (N) and property (N^c)?

The only known example of such a graph is the countable *random graph*, or *Rado's graph*, the unique countable existentially closed graph. However, there are $2^{\aleph_0}$ non-isomorphic graphs having one of these properties.

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