Queen Mary University of London

MTHM024/MTH714U — Group Theory

Problem Sheet 4 — **3 November 2011**

The problem sheets in this course are for "formative assessment" only; there is no coursework component in the assessment.

Any work handed in by the lecture on the date at the top of the sheet will be marked and returned to you in the next week's lecture.

1 (a) Show that every subgroup of a cyclic group is cyclic.

(b) Show that every subgroup of a dihedral group is cyclic or dihedral.

(c) Let G be the dihedral group of order 12. Find all subgroups of G, indicating which are normal.

(d) Find all the composition series for the dihedral group of order 12.

2 Find all the composition series for the symmetric group S_4.

3 Let G be a group of order 120, whose composition factors are C_2 and A_5.

(a) Show that, if G has more than one composition series, then $G \cong C_2 \times A_5$.

(b) Find such a group whose unique composition series $1 \lhd H \lhd G$ has $|H| = 60$.

(c) (Harder) Find such a group whose unique composition series $1 \lhd H \lhd G$ has $|H| = 2$.

4 Let p be a prime number. An *elementary abelian p-group* is a group G such that G is abelian and $g^p = 1$ for all $g \in G$.

(a) Show that an elementary abelian p-group has order a power of p.

(b) Show that if G is an elementary abelian group of order p^n, then

$$G \cong C_p \times C_p \times \cdots \times C_p \qquad (n \text{ factors}).$$