1 (a) Show that every subgroup of a cyclic group is cyclic.

(b) Show that every subgroup of a dihedral group is cyclic or dihedral.

(c) Let $G$ be the dihedral group of order 12. Find all subgroups of $G$, indicating which are normal.

(d) Find all the composition series for the dihedral group of order 12.

2 Find all the composition series for the symmetric group $S_4$.

3 Let $G$ be a group of order 120, whose composition factors are $C_2$ and $A_5$.

(a) Show that, if $G$ has more than one composition series, then $G \cong C_2 \times A_5$.

(b) Find such a group whose unique composition series $1 \triangleleft H \triangleleft G$ has $|H| = 60$.

(c) (Harder) Find such a group whose unique composition series $1 \triangleleft H \triangleleft G$ has $|H| = 2$.

4 Let $p$ be a prime number. An **elementary abelian $p$-group** is a group $G$ such that $G$ is abelian and $g^p = 1$ for all $g \in G$.

(a) Show that an elementary abelian $p$-group has order a power of $p$.

(b) Show that if $G$ is an elementary abelian group of order $p^n$, then

$$G \cong C_p \times C_p \times \cdots \times C_p \quad (n \text{ factors}).$$