Queen Mary University of London

MTHM024/MTH714U **Group Theory**

Problem Sheet 2 **27 October 2011**

The problem sheets in this course are for "formative assessment" only; there is no coursework component in the assessment.

Any work handed in by the lecture on the date at the top of the sheet will be marked and returned to you in the next week's lecture.

1 A *transposition* is a permutation which interchanges two points and fixes the others.

(a) Show that the symmetric group S_n is generated by its transpositions for $n \geq 2$.

(b) Let G be a subgroup of S_n containing a transposition. Define a relation $\sim$ on the set $\{1,2,\ldots,n\}$ by the rule that $i \sim j$ if either $i = j$ or the transposition (i, j) belongs to G. Prove that $\sim$ is an equivalence relation. Show that the transpositions contained in any equivalence class generate the symmetric group on that class.

(c) Hence show that G has a normal subgroup which is the direct product of symmetric groups on the equivalence classes of $\sim$.

2 Let G be the symmetric group S_5.

(a) For each prime p dividing $|G|$, find a Sylow p-subgroup of G and determine its structure; find also the number of Sylow p-subgroups.

(b) FInd all the normal subgroups of G.

3 (a) Show that a group of order 40 has a normal Sylow subgroup.

(b) Do the same for a group of order 84.