1 The figure below is the Fano plane, a configuration of seven points and seven lines. A symmetry is a permutation of the seven points which carries lines to lines.

(a) Let \( a, b, c \) and \( A, B, C \) be two triples of distinct points, neither of which forms a line. Show that there is a unique symmetry of the Fano plane carrying the first to the second.

(b) Show that the symmetries of the Fano plane form a group \( G \) of order 168.

(c) Describe the Sylow subgroups of \( G \).

(d) Show that \( G \) is simple.