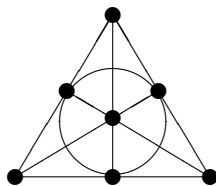


---

*The problem sheets in this course are for “formative assessment” only; there is no coursework component in the assessment.*

*Any work handed in by the lecture on the date at the top of the sheet will be marked and returned to you in the next week’s lecture.*

- 1** The figure below is the *Fano plane*, a configuration of seven points and seven lines. A *symmetry* is a permutation of the seven points which carries lines to lines.



- (a) Let  $a, b, c$  and  $A, B, C$  be two triples of distinct points, neither of which forms a line. Show that there is a unique symmetry of the Fano plane carrying the first to the second.
- (b) Show that the symmetries of the Fano plane form a group  $G$  of order 168.
- (c) Describe the Sylow subgroups of  $G$ .
- (d) Show that  $G$  is simple.