

MTH5122 Statistical Methods

Assignment 2

Feedback question to be handed in by Wednesday, 9th October, 3.00 pm. Please put the room and time of the tutorial you are attending so the work can be returned to you. Please put your assignment into the blue box in the Basement of the Maths Building. The work should be stapled or put in a folder.

1. Let X have the pdf $f(x) = 5x^4$, $0 < x < 1$ and zero otherwise. Find the pdf of $Y = 2X - 1$.
2. Let X have the pdf $f(x) = x \exp(-\frac{x^2}{2})$ $0 < x < \infty$ and zero otherwise. Find the pdf of $Y = X^2$.
3. The pdf of X is $f(x) = \theta x^{-(\theta+1)}$, $x > 1$ and 0 otherwise, where θ is a positive parameter. Let $Y = \ln X$. Find the distribution of Y .
4. The random variables X and Y have a joint pdf given by

$$f_{X,Y}(x, y) = 6xy^2 \quad 0 < x < 1, 0 < y < 1$$

and zero otherwise. Find the cdf of $U = X + Y$ and hence its pdf.

(Hint: follow the same steps I did for the uniform distribution over the unit square.)

5. FEEDBACK QUESTION

- (a) Suppose the random variable W is uniformly distributed on the interval $(-\pi/2, \pi/2)$.
 - i. Find the pdf and cdf of W .
 - ii. Find the cdf of $X = \tan W$.
 - iii. Hence find the pdf of X .
- (b) The joint probability density function of X_1 and X_2 is given by

$$f_{X_1, X_2}(x_1, x_2) = Cx_1x_2^3 \quad 0 < x_1 < x_2 < 1$$

and zero otherwise.

- i. Find C .
- ii. Find the marginal distribution of X_2 .
- iii. Suppose now that $Y_1 = X_1/X_2$ and $Y_2 = X_2$.
 - A. Find the joint probability density function of Y_1 and Y_2 .
 - B. Hence find the marginal probability density function of Y_1 .