Queen Mary
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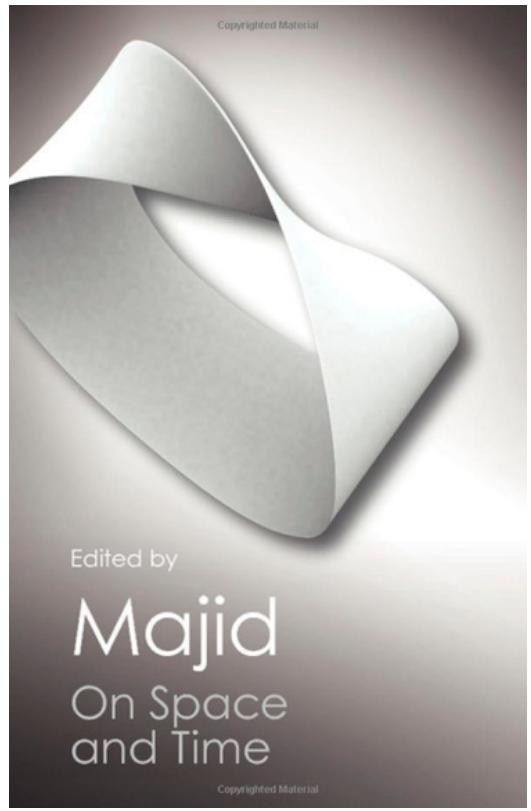
- MSc maths, financial maths
- PhD research programmes
- East London location

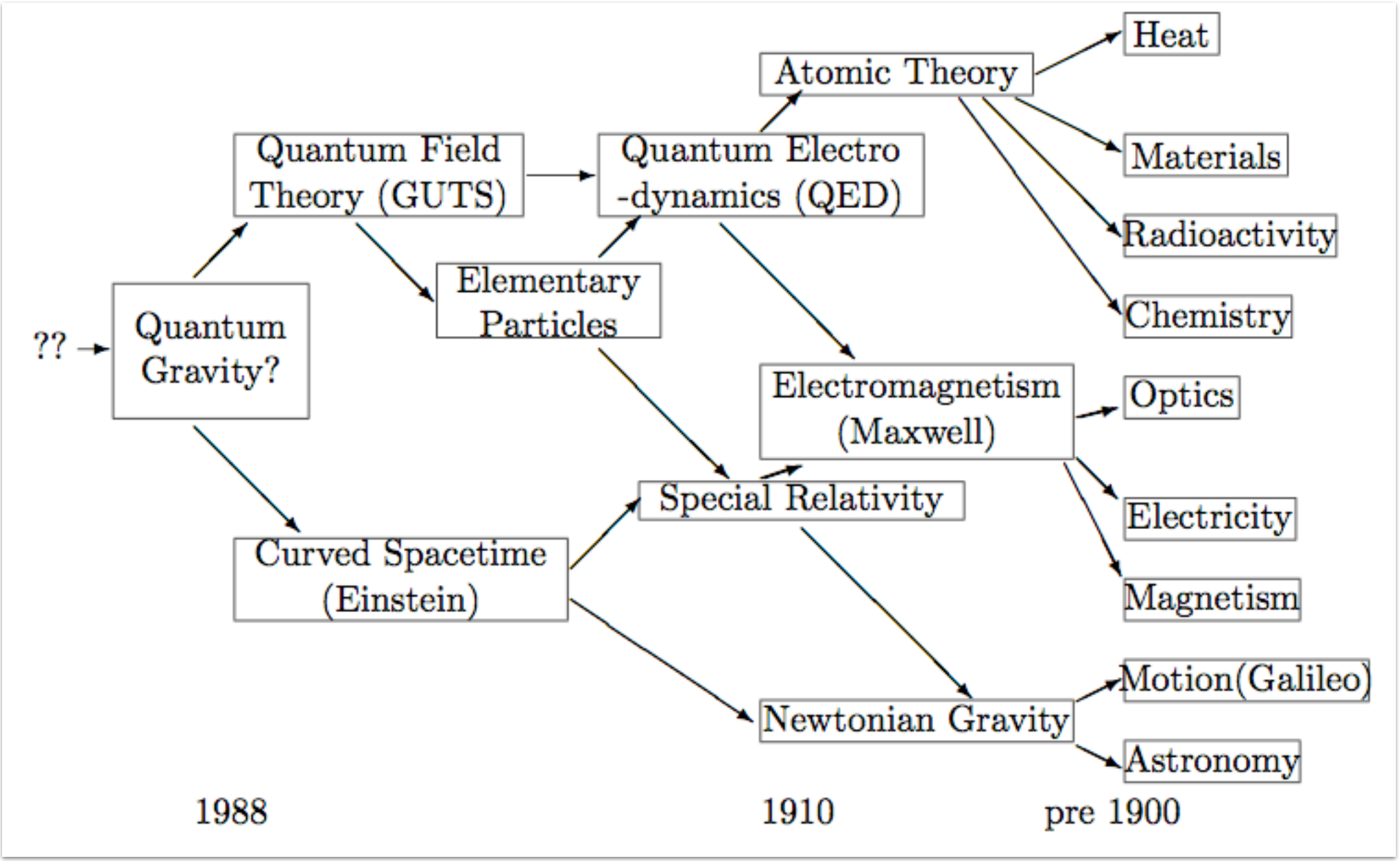
Representation and Reality

- Is there an absolute reality 'Nature'?
- Is our world a representation or shadow of a Platonic reality?
- or are we inside a cosmic simulation as in 'The Matrix'?

In my book I propose a new observer-observed dualism inspired by pure mathematics as a new foundation for Quantum Gravity.



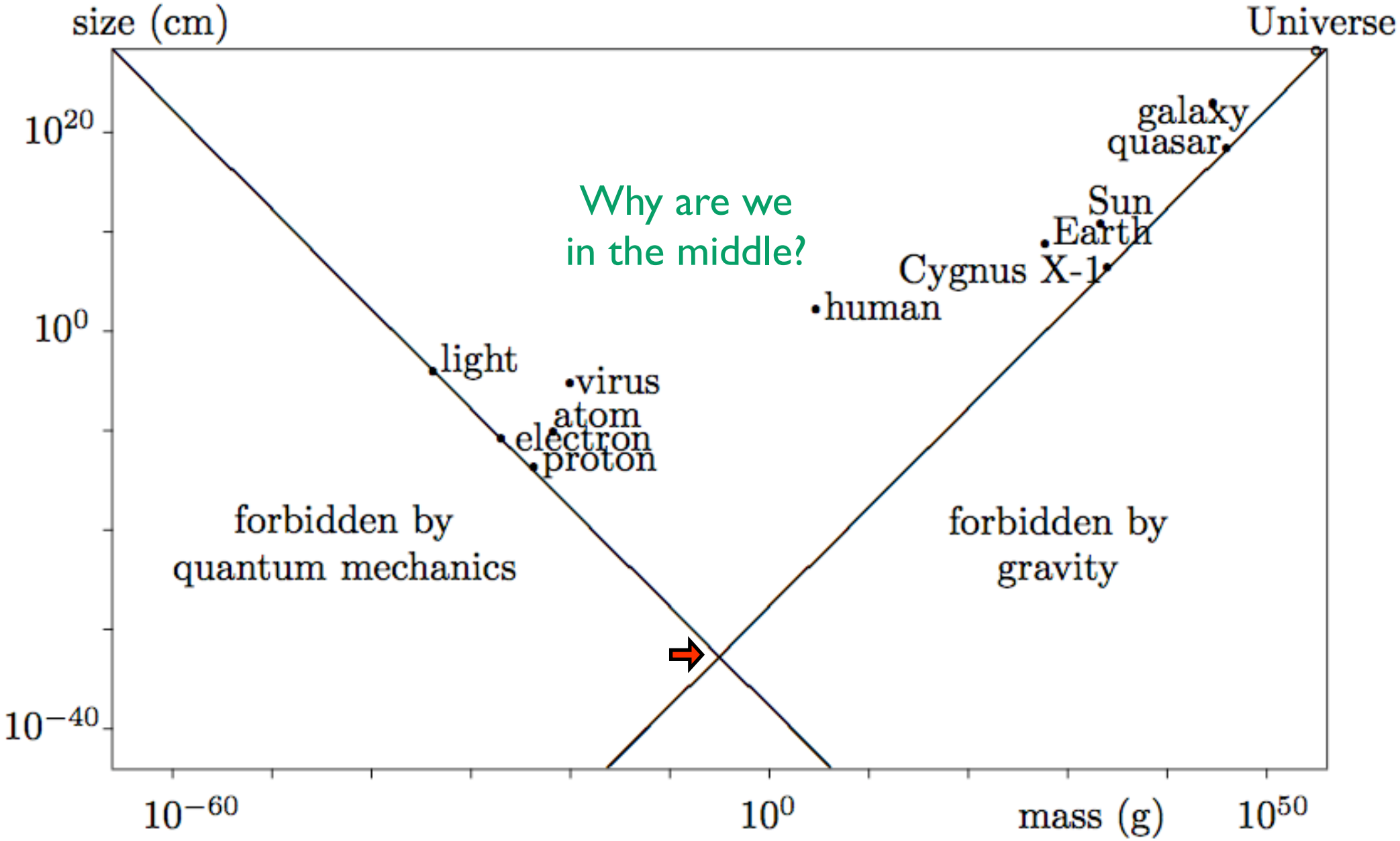
Are we in sight of the end of physics?



If so, what is the essential nature of physical reality?

The big picture

$10^{10} = 10,000,000,000$
$10^{-10} = 0.0000000001$



Planck scale = $2 \times 10^{-5}\mathrm{g}, \quad 1.6 \times 10^{-33}\mathrm{cm}$

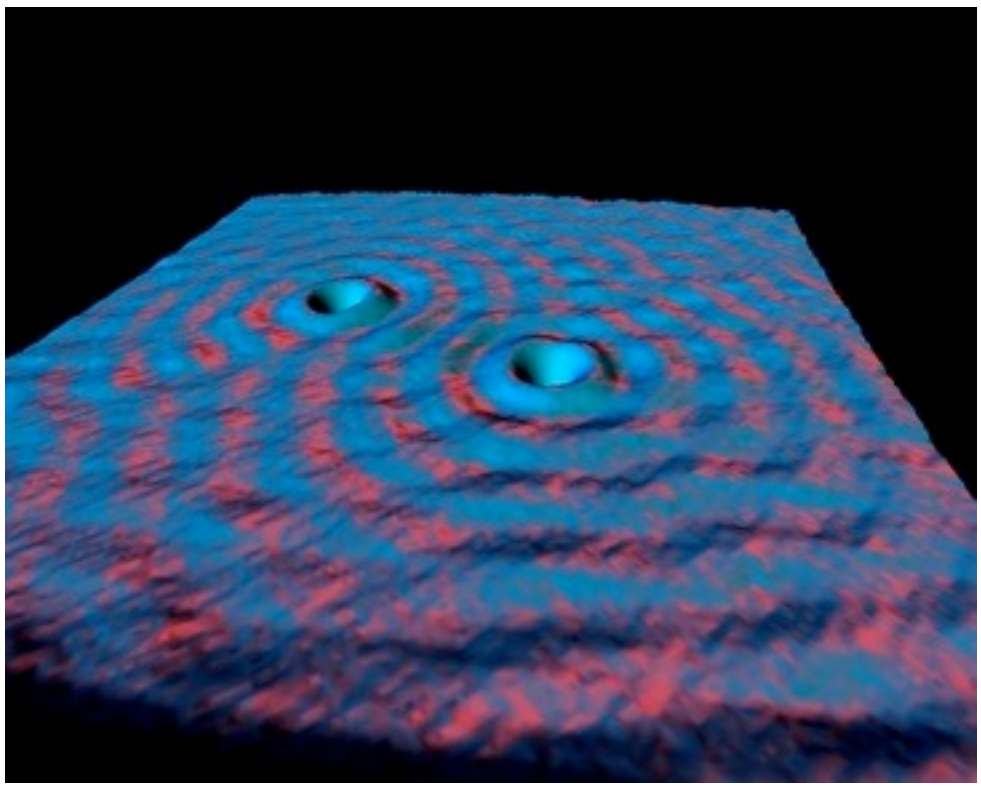
Quantum theory

Light of wavelength λ has energy $E = \hbar \frac{c}{\lambda}$

(discovered by Planck, $\hbar = 6.6 \times 10^{-34} \mathrm{Js}$)

Energy and mass are interchangeable by $E = mc^2$

(discovered by Einstein, $c = 3 \times 10^{10} \mathrm{cm/s}$ speed of light)



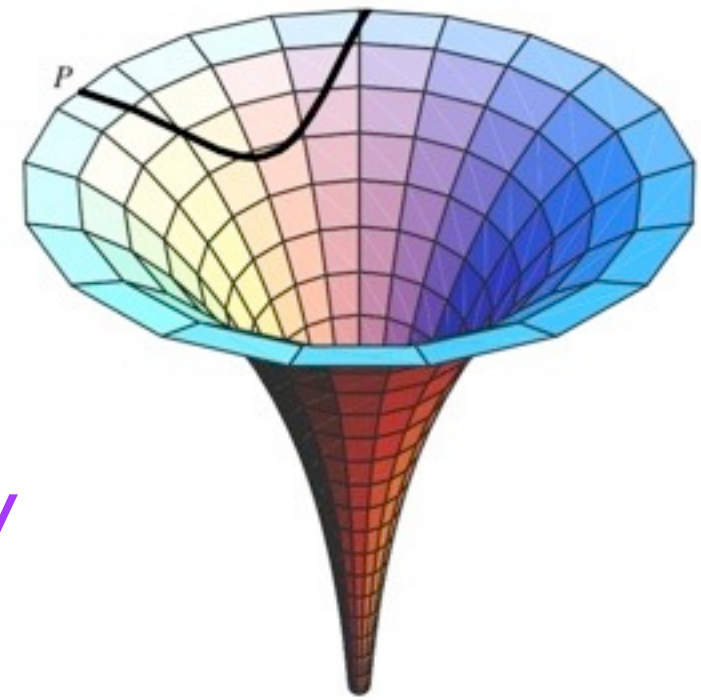
$$\lambda = \frac{\hbar}{mc}$$

This formula works for all kinds of particle - waves

The image on the left is an electron wave scattering off atomic defects in a copper crystal

Gravity (curved spacetime)

Surface of constant negative curvature. Ants moving from *P* on what at each point is for them a straight line would be deflected *outwards*



Discovered by Einstein: Curvature of 4-dimensional spacetime *is* gravity

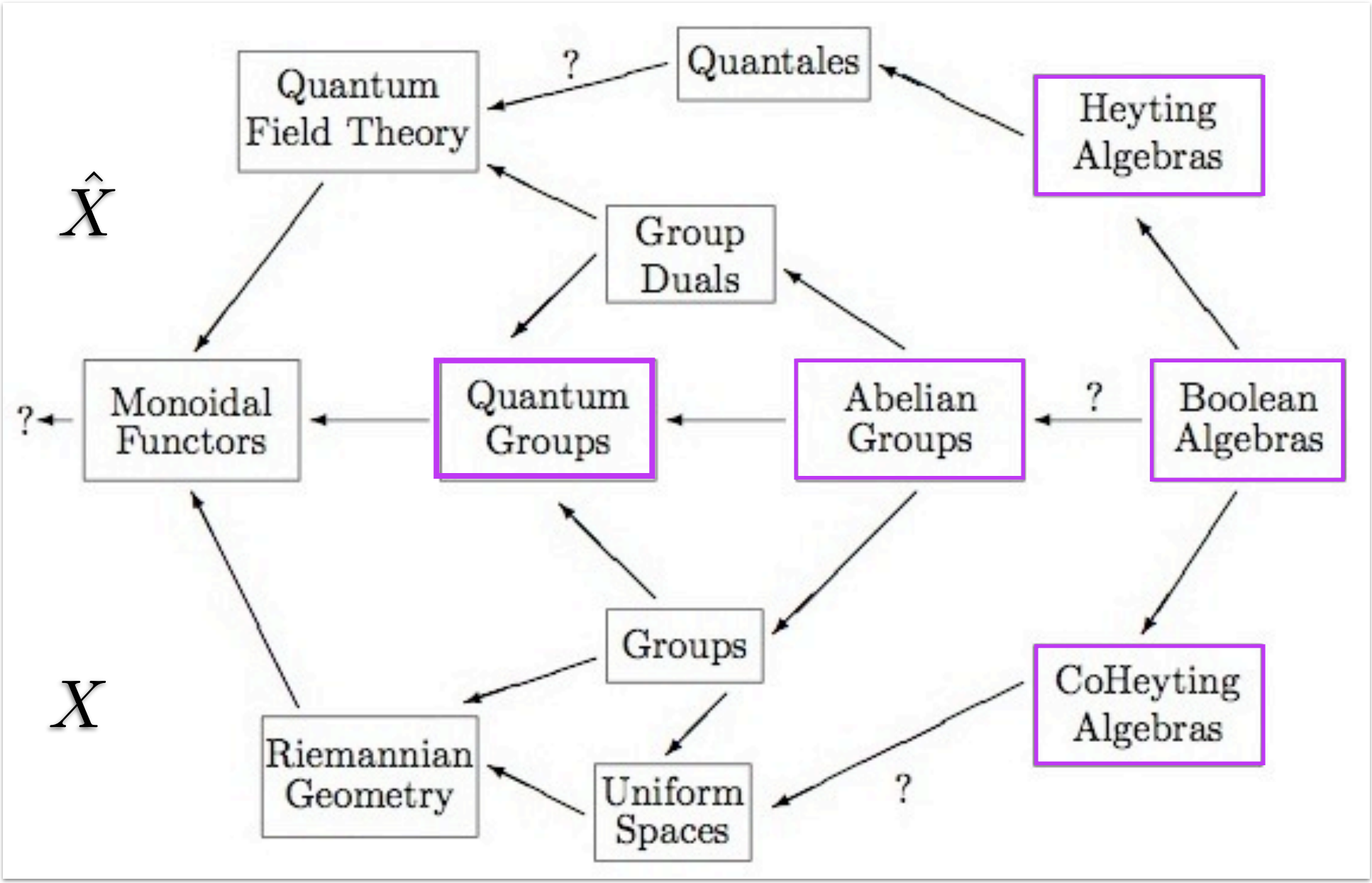
⇨ Black hole of mass M has size

$$r = \frac{GM}{c^2}$$

($G = 10^{-8}\mathrm{cm}^3/\mathrm{gs}^2$ Newton's constant)

Artists impression of a black hole. Our own galaxy has giant black hole in its centre

Compare with mathematics of self-duality



Postulate: the search for the ultimate theory of physics *is* the search for a self-dual structure in a self-dual category

Boolean algebra

Simplest theory of Physics is Logic: organise the world into subsets $A \subseteq E$ of `things' taken from a universe E

Algebra of union $\cup$ and intersection $\cap$ of subsets

$$A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$A \cup \text{not}A = E$ (everything) `either something is true or its opposite is'

$A \cap \text{not}A = \emptyset$ (nothing) `nothing is true and not true'

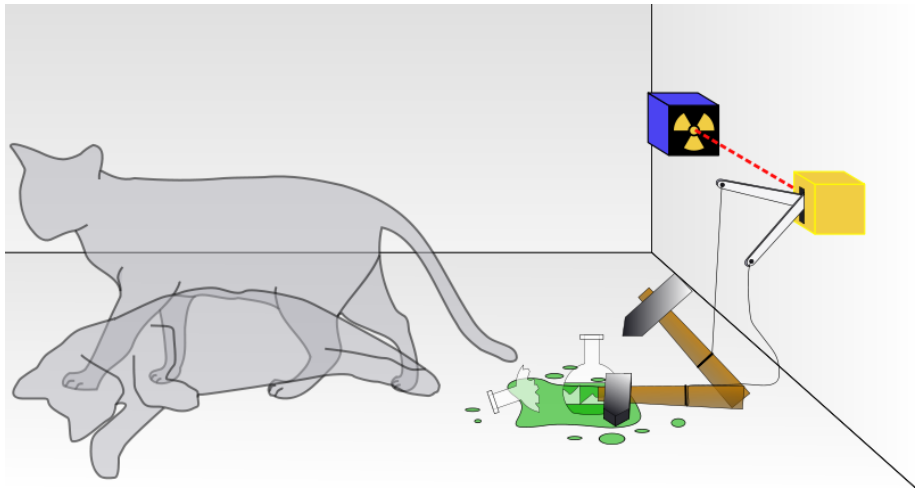
De Morgan Duality

$A \leftrightarrow \text{not}A \qquad \cap \leftrightarrow \cup \qquad (\text{i.e. and} \leftrightarrow \text{or}) \qquad \text{everything} \leftrightarrow \text{nothing}$

$$\text{not}(A \cap B) = (\text{not } A) \cup (\text{not } B)$$

Quantum Logic (Heyting algebra): relax the axiom

$$A \cup \text{not} A \neq \text{everything}$$



Famous example Schroedingers cat: radioactive decay releases poison gas. From outside the room cat is neither dead nor alive

this is the birth of quantum theory

Dual theory (coHeyting algebra): relax the axiom

$$A \cap \text{not} A \neq \text{nothing}$$

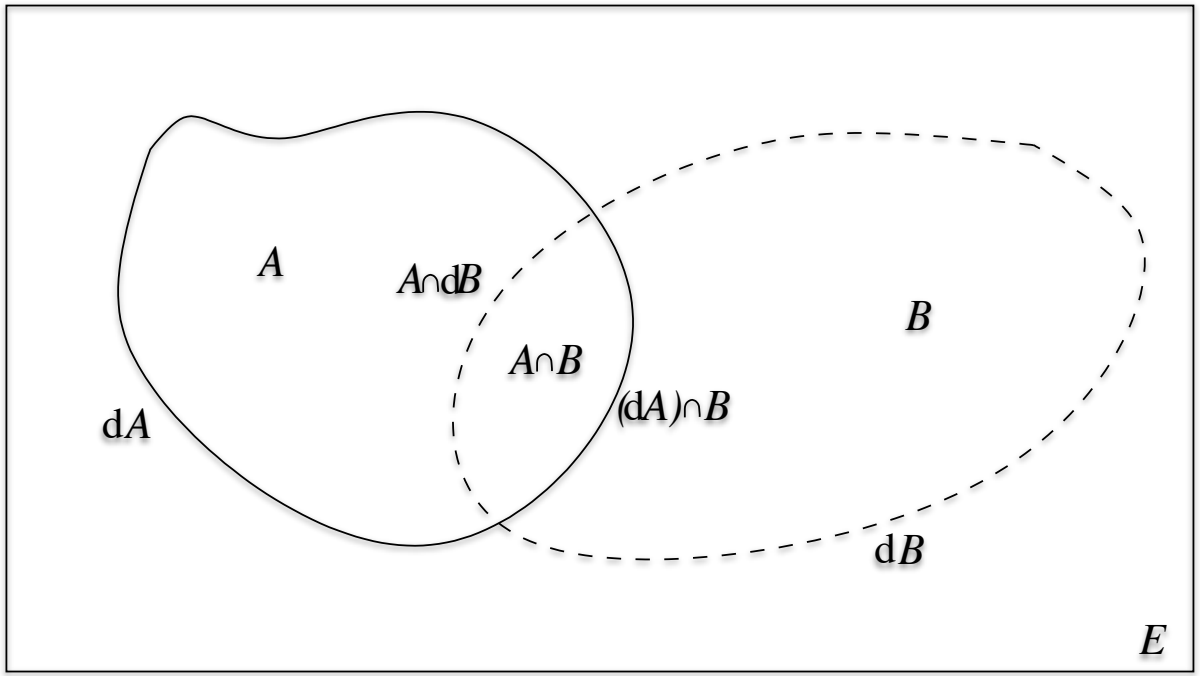
this is the birth of something else: I claim its the birth of differential geometry/gravity

Define $\mathrm{d}(A) = A \cap \text{not } A$ `The boundary of A'

This obeys the same `product rule' $\mathrm{d}(ab) = (\mathrm{d}a)b + a\mathrm{d}b$ as for differentiation, namely

$$\mathrm{d}(A \cap B) = ((\mathrm{d}A) \cap B) \cup (A \cap \mathrm{d}B)$$

Sketch of Proof:

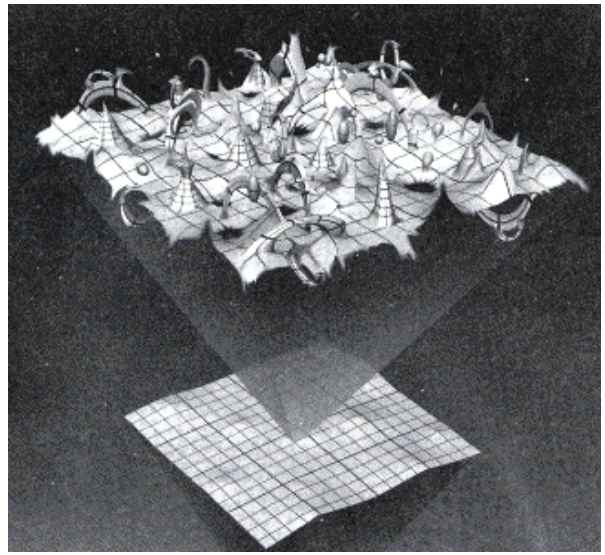


Falling into a black hole (Dual of Schroedingers cat): from afar you never fall in because of infinite time dilation but from your point of view you do, and you die. So you are both dead and alive

Restoration of De Morgan duality in quantum gravity!

Apples curve space, not-apples do not. De Morgan duality is not a property of gravity alone, nor of quantum theory alone.

$\longleftrightarrow$



`this space is as full of apples as gravity allows'

`this space is as empty of not-apples as quantum theory allows'

black hole $\longleftrightarrow$ vacuum energy

(dark energy? $10^{-29}\mathrm{g/cm}^3$)

Fourier duality for Abelian groups

G set with $\cdot$, $(\ )^{-1}$

$x \cdot y = y \cdot x$

$\hat{G}$ set of maps $\phi : G \to \mathbb{C}$

$\phi(x \cdot y) = \phi(x)\phi(y)$

- $\hat{G}$ is itself an Abelian group $(\phi \cdot \psi)(x) = \phi(x)\psi(x)$
- $\hat{\hat{G}} \cong G$ so its symmetrical
- Fourier theory is just an extension of the duality between $G, \hat{G}$

$$\mathcal{F} : C(G) \to C(\hat{G})$$

$$\mathcal{F}(f)(\phi) = \sum_{x \in G} \phi(x) f(x)$$

eg $f \in C(\mathbb{R}^3)$

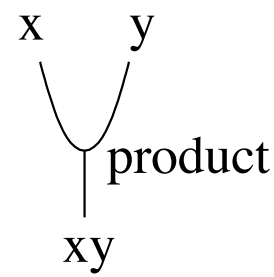
$x \in \mathbb{R}^3$

$$\mathcal{F}(f)(p) = \int \mathrm{d}x e^{\imath x \cdot p} f(x)$$

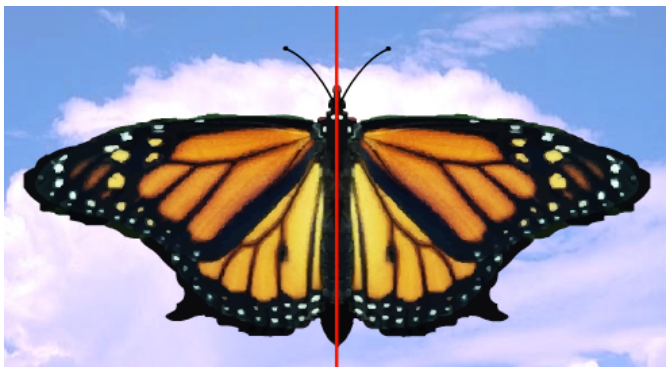
$p \in \hat{\mathbb{R}}^3 \cong \mathbb{R}^3$ labels the representations

Quantum groups

An algebra takes two things and multiplies them



So we also want to be able to `unmultiply'



Already implicit in the ancient greek notion of symmetry in `xerox map'

flip(x R y)= flip(x) R flip(y)

flip, rotation, other symmetries form a group G,

$$\Delta x = x \otimes x$$

(an ordinary group viewed trivially as a quantum group)

Definition: a quantum group is

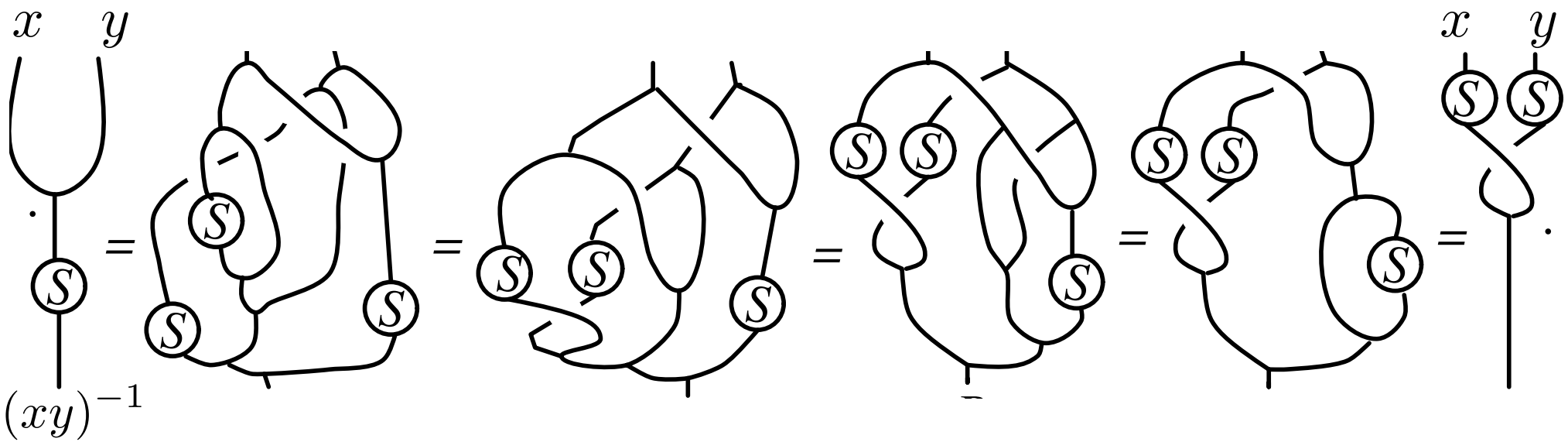
$\cdot : H \otimes H \to H$ product

$\Delta : H \to H \otimes H$ coproduct

$S : H \to H$ antipode or `inverse'

genuine examples first emerged in the 1980s

- Most of classical group theory still goes through. Eg analogous to the proof that $(xy)^{-1} = y^{-1}x^{-1}$ we have



This algebra is like wiring up a computer except

- information flows down the page
- it matters if a `wire' jumps under or over another one

→ Quantum groups also have applications knot theory & possibly quantum computers

Quantum group duality

H^* the set of maps $\phi : H \to \mathbb{C}$ is the dual quantum group

(Proof: the transpose of a map $V \to W$ is a map $W^* \to V^*$)

$$[x, p] = \imath\hbar(1 - e^{-\frac{x}{r}})$$

$$\Delta x = x \otimes 1 + 1 \otimes x \qquad \Delta p = p \otimes e^{-\frac{x}{r}} + 1 \otimes p$$

is a self-dual quantum group, $H \cong H^*$

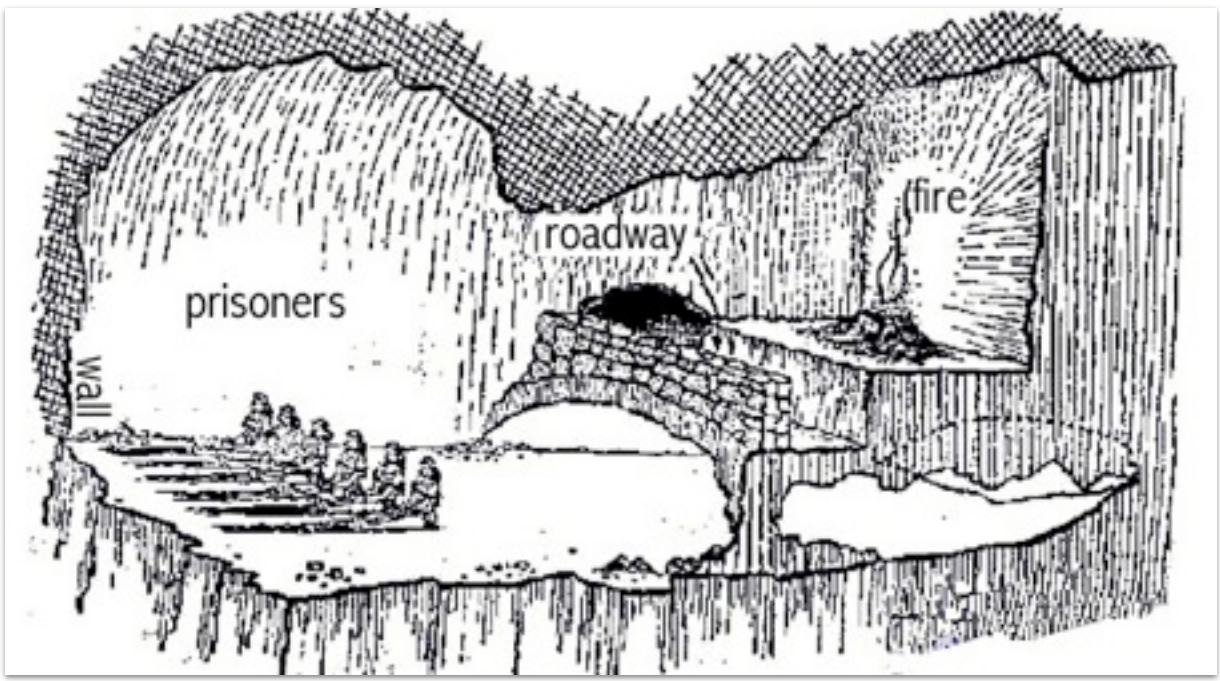
- A particle approaching 0 from above slows down and takes ∞ time to reach it ⇨ interpret r as gravity
- The famous `Heisenberg algebra' $[x, p] = \imath\hbar$ is not a quantum group, need both quantum and gravity, their role swapped by the duality.

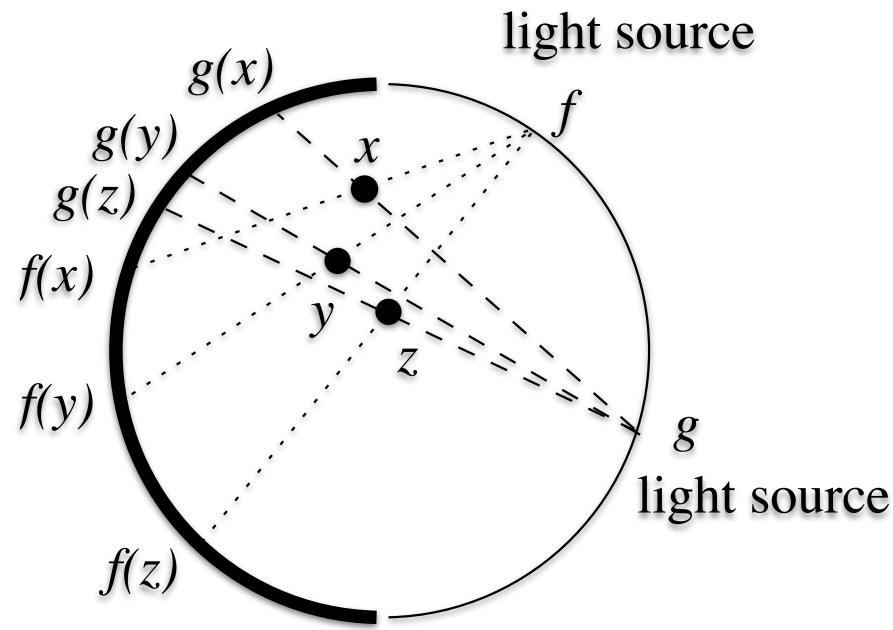
$$f(x) = x(f)$$ or my answer to Plato's cave

When you measure the value of f at point x in X, you could equally interpret it as measuring the value of x at the point f in $\hat{X}$

$$\hat{\hat{X}} \cong X$$

The set $\hat{X}$ of representations by different light positions f is as real as the arrangement X of objects x on the roadway





17th century Scientific Revolution

replaced God as source of reality by ideas going back to greek, latin and arabic thinkers: Scientific Method provided rules of engagement between 'Nature' and 'experiment'.

But if X is 'real' then equally is $\hat{X}$ and we should accept both or look for a self-dual object where $\hat{X} \cong X$

Bishop Berkeley (1686-1753)

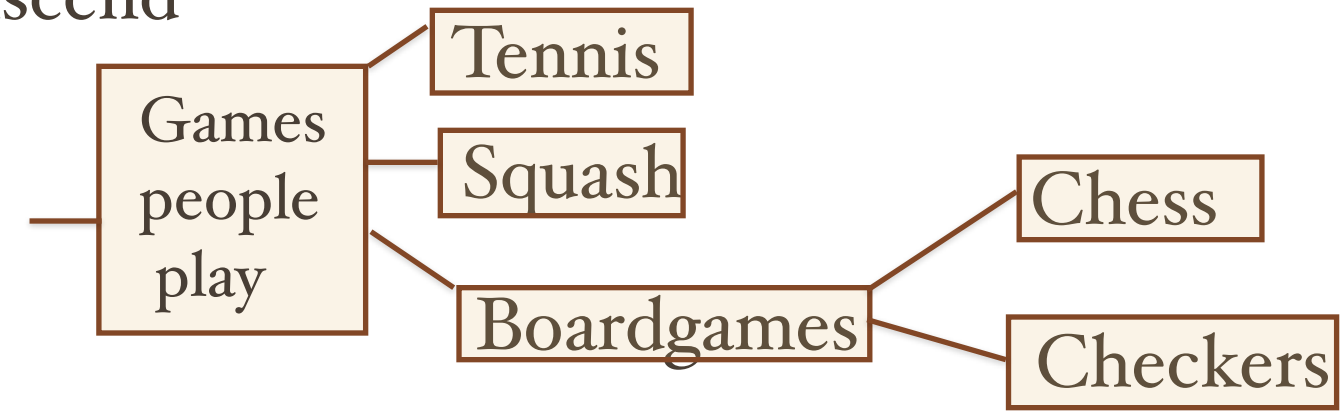
"If there was no other matter in the Universe then the water in a rotating bucket would stay flat" - cf Mach, Einstein

It seems that the *structure* of physics (to which just add colourful names) is determined by the way of looking at the world called being a physicist. We are rediscovering our own assumptions

Relative Realism

As in pure mathematics or the game of chess, reality is *created by the choice* to work within certain rules. But the menu of possible choices is *out there* (apparently absolute) ... but created by higher level choices

Reality is both absolute *and* created by assumptions which we can transcend



- If enough for Physics, maybe all there is!
- We can still have the rigidity of science
- Unlike Buddhism, nexus of possible choices is not arbitrary, but we can navigate in it

Corealistic Rocker by Friedrich Kiesler